Er ligningssystemet konsistent? Løs ligningssystemet.

$$\begin{array}{rcrcr} x_1 & - & 2x_2 & = & -6 \\ -2x_1 & + & 3x_2 & = & 7 \end{array}$$

totalmatrix:

$$[A \mid \vec{b}] = \left[\begin{array}{cc|c} 1 & -2 & -6 \\ -2 & 3 & 7 \end{array}\right]$$

$$\underset{R_2 \to R_2 + 2R_1}{\sim} \left[\begin{array}{cc|c} 1 & -2 & -6 \\ 0 & -7 & -5 \end{array}\right] = T$$

, T har ingen pivot i sidste søjle

$\Rightarrow$ systemet er konsistent!

$$R_2 \to -R_2 \quad \sim \left[\begin{array}{cc|c} 1 & -2 & -6 \\ 0 & 1 & 5 \end{array}\right] \quad R_1 \to R_1 + 2R_2 \quad \sim \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 5 \end{array}\right] = [R|\vec{c}\,]$$

$[R|\vec{c}\,]$ er totalmatrix for ligningsystemet

$$\begin{cases} 1 \cdot x_1 + 0 \cdot x_2 = 4 \\ 0 \cdot x_1 + 1 \cdot x_2 = 5 \end{cases}$$

$$= \begin{matrix} x_1 = 4 \\ x_2 = 5 \end{matrix}$$

$$x_1 + (-2)x_2 \overset{?}{=} -6 \qquad 4 + (-2)\cdot 5 = -6 \quad \checkmark$$

$$-2x_1 + 3x_2 \overset{?}{=} 7 \qquad -2\cdot 4 + 3\cdot 5 = 7 \quad \checkmark$$

Løsning af ligningssystem

~~(Kapitel 1.4 eksempel 1)~~ Er ligningssystemet konsistent? Løs ligningssystemet.

$$\begin{array}{rcrcrcrcrcl}
x_1 & + & 2x_2 & - & x_3 & + & 2x_4 & + & x_5 & = & 2 \\
-x_1 & - & 2x_2 & + & x_3 & + & 2x_4 & + & 5x_5 & = & 6 \\
2x_1 & + & 4x_2 & - & 3x_3 & + & 2x_4 & + & & = & 3 \\
-3x_1 & - & 6x_2 & + & 2x_3 & + & & & 3x_5 & = & 9
\end{array}$$

totalmatrix:

$$\left[\begin{array}{ccccc|c}
1 & 2 & -1 & 2 & 1 & 2 \\
-1 & -2 & 1 & 2 & 5 & 6 \\
2 & 4 & -3 & 2 & 0 & 3 \\
-3 & -6 & 2 & 0 & 3 & 9
\end{array}\right]
\begin{array}{c}
R_2 \to R_2 + R_1 \\
R_3 \to R_3 - 2R_1 \\
\sim \\
R_4 \to R_4 + 3R_1
\end{array}
\left[\begin{array}{ccccc|c}
1 & 2 & -1 & 2 & 1 & 2 \\
0 & 0 & 0 & 4 & 6 & 8 \\
0 & 0 & -1 & -2 & -2 & -1 \\
0 & 0 & -1 & 6 & 6 & 15
\end{array}\right]$$

$$\overset{R_2 \leftrightarrow R_3}{\sim} \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & -2 & -2 & -1 \\ 0 & 0 & 0 & 4 & 6 & 8 \\ 0 & 0 & -1 & 6 & 6 & 16 \end{array}\right] \overset{R_4 \to R_4 - R_2}{\sim} \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & -2 & -2 & -1 \\ 0 & 0 & 0 & 4 & 6 & 8 \\ 0 & 0 & 0 & 8 & 8 & 16 \end{array}\right]$$

$$\overset{R_3 \to \frac{R_3}{2}}{\sim} \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & -2 & -2 & -1 \\ 0 & 0 & 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 8 & 8 & 16 \end{array}\right] \overset{R_4 \to R_4 - 4R_3}{\sim} \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & -2 & -2 & -1 \\ 0 & 0 & 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & -4 & 0 \end{array}\right]$$

T har ingen pivot i sidste søjle $\Rightarrow$ systemet konsistent

$$R_4 = \left(-\tfrac{1}{4}R_4\right) \sim \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 2 & 1 & 2 \\ 0 & 0 & -1 & -2 & -2 & -1 \\ 0 & 0 & 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \underset{\substack{R_3 \to R_3 - 3R_4 \\ R_2 \to R_2 + 2R_4 \\ R_1 \to R_1 - R_4}}{\sim} \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 2 & 0 & 2 \\ 0 & 0 & -1 & -2 & 0 & -1 \\ 0 & 0 & 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\underset{\substack{R_1 \to R_1 - R_3 \\ R_2 \to R_2 + R_3}}{\sim} \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 0 & 0 & -2 \\ 0 & 0 & -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \underset{R_3 = \frac{R_3}{2}}{\sim} \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 0 & 0 & -2 \\ 0 & 0 & -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to -R_2} \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \underset{R_1 \to R_1 + R_2}{\sim} \begin{array}{c} \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \\ \begin{array}{cccccc} 1 & & 3 & 4 & 5 & \end{array} \end{array}$$

$$\sim$$

R har pivot i søjle 1, 3, 4, 5, $\quad [R \mid \vec{c}\,]$

har <u>ingen</u> pivot i søjle 2 $\Rightarrow$ x_2 fri variabel

$$\begin{aligned} x_1 + 2x_2 + 0x_3 + 0x_4 + 0x_5 &= -5 \\ +1x_3 + \qquad\qquad &= -3 \\ x_4 \qquad &= 2 \\ x_5 &= 0 \end{aligned}$$

$$\begin{cases} x_1 = -5 - 2x_2 \\ x_3 = -3 \\ x_4 = 2 \\ x_5 = 0 \end{cases}, \quad \{ x_2 = s \text{ (parameter)}$$

$$\begin{aligned} x_1 &= -5 - 2s \\ x_2 &= 0 + 1 \cdot s \\ x_3 &= -3 + 0 \cdot s \\ x_4 &= 2 + 0 \cdot s \\ x_5 &= 0 + 0 \cdot s \end{aligned}, \quad s \in \mathbb{R} \Rightarrow$$

vektorform $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ -3 \\ 2 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$s = \mathbb{R}$.

Udspændende vektorer

Spænd

Las $S = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ være vektorer i $\mathbb{R}^n$. Spændet af S defineres som

$$\text{span}(S) := \{c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_k\vec{v}_k \mid c_1, c_2, \ldots, c_k \in \mathbb{R}\}.$$

(Mængden af *alle mulige* linearkombinationer af vektorer fra S)

Ækvivalente betingelser

Betragt vektorerne $S = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ i $\mathbb{R}^m$, og lad $A = [\vec{v}_1 \vec{v}_2 \cdots \vec{v}_k]$. Følgende betingelser er ækvivalente

- Vektorerne i S udspænder $\mathbb{R}^m$
- $A\vec{x} = \vec{b}$ er konsistent for alle $\vec{b} \in \mathbb{R}^m$
- $A \sim R$, hvor R har pivot i hver række. ($\text{rank}(A) = m$).

Er $\vec{v}$ i span(S)?

$S = \left\{ \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \end{bmatrix} \right\}$. Er $\begin{bmatrix} 8 \\ -4 \end{bmatrix}$ i span (S), ?

Er $\vec{v}(r)$ i span(S), r parameter

$S = \left\{ \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \end{bmatrix} \right\}$. For hvilke r er $\begin{bmatrix} 2 \\ r \end{bmatrix}$ i span (S), ?