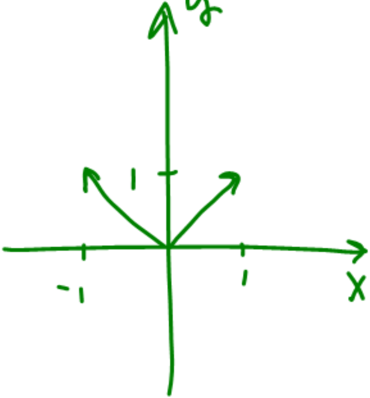
Vektorrum eksempler:

	$\mathbb{R}^2$	$\mathbb{R}^3$
Basis	$\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$ $= \{\vec{e}_1, \vec{e}_2\}$	$\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$ $= \{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$
Dimension	2	3

$\mathbb{R}^2$

Basis $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$



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ex $\mathbb{R}^3$

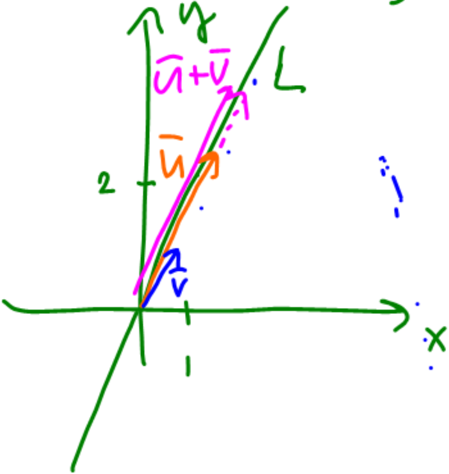
Basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

Underrum af $\mathbb{R}^2$

$$L = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2, \text{ hvor } y = 2x \right\}$$

i) $\vec{u}, \vec{v} \in L$

$\vec{u} + \vec{v} \in L$



ii) $\alpha \in \mathbb{R}$

(ex $\alpha = 4$)

$\alpha \vec{v} \in L$

$$L = \left\{ s\begin{bmatrix}1\\2\end{bmatrix},\ s \in \mathbb{R} \right\}$$

$$= \operatorname{span}\left(\left\{\begin{bmatrix}1\\2\end{bmatrix}\right\}\right)$$

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x\\2x\end{bmatrix} = x\begin{bmatrix}1\\2\end{bmatrix}$$

$$x \to s$$

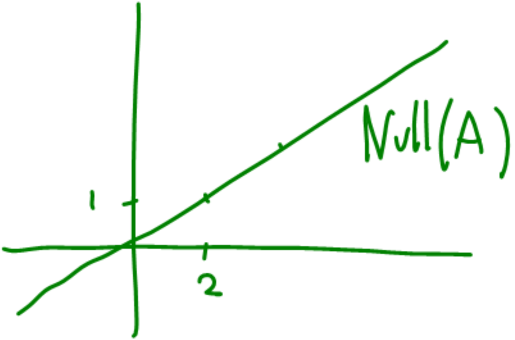
$$A = \begin{bmatrix}1 & -2\\0 & 0\end{bmatrix}$$

$$\operatorname{Null}(A) = \left\{ \begin{bmatrix}x_1\\x_2\end{bmatrix} : A\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}1 & -2\\0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \vec{0} \right\}$$

$$\begin{array}{cc}\uparrow & \uparrow\\ P & \neg P\end{array}$$

$$= \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : 1 \cdot x_1 - 2x_2 = 0, \ \cancel{0 \cdot x_1 + 0 x_2 = 0} \right\}$$

$$= \left\{ \begin{bmatrix} x \\ y \end{bmatrix} ; \ x = 2y \Leftrightarrow y = \frac{1}{2} x \right\}$$



$x_1 = 2x_2$, x_2 fri

$\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$ er i trappefor, pivot i 1ste søjle

÷ pivot i 2nd søjle

$$x_1 = 2x_2 = 2s$$
$$x_2 = s = 1 \cdot s$$

$$\left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = s \begin{bmatrix} 2 \\ 1 \end{bmatrix} , s \in \mathbb{R} \right\} = \text{alle løsn. til } A\vec{x} = \vec{0}$$

$$= \mathrm{Null}(A)$$

$$= \mathrm{span}\left(\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\} \right)$$

Koordinater

Definition 4: Koordinatvektor

Lad $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_p\}$ være en basis for et underrum V af $\mathbb{R}^n$. **Koordinatvektoren** for $\vec{x} \in V$ relativt til $\mathcal{B}$ er givet ved

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_p \end{bmatrix}, \qquad \text{hvor } \vec{x} = c_1\vec{b}_1 + c_2\vec{b}_2 + \cdots + c_p\vec{b}_p.$$

Bemærkning: Koordinatvektoren for $\vec{x} \in V$ er den *entydige løsning* til ligningssystemet

$$\left[\vec{b}_1 \; \vec{b}_2 \; \cdots \; \vec{b}_p \,|\, \vec{x}\right].$$

Specialtilfælde: Hvis $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_n\}$ er en basis for $\mathbb{R}^n$, så er $n \times n$-matricen $B = [\vec{b}_1 \; \vec{b}_2 \; \cdots \; \vec{b}_n]$ invertibel, og vi har for $\vec{x} \in \mathbb{R}^n$,

$$[\vec{x}]_{\mathcal{B}} = B^{-1}\vec{x}, \qquad \text{da } B[\vec{x}]_{\mathcal{B}} = \vec{x}.$$

(ex) $\mathcal{B} = \{\bar{b}_1, \bar{b}_2, \bar{b}_3\}$ basis for $V \in \mathbb{R}^5$

$\vec{x}$ er givet

hvis $\vec{x} = c_1\bar{b}_1 + c_2\bar{b}_2 + c_3\bar{b}_3$

<u>og</u> $\vec{x} = d_1\bar{b}_1 + d_2\bar{b}_2 + d_3\bar{b}_3$

$$\vec{x} - \vec{x} = (c_1\bar{b}_1 + c_2\bar{b}_2 + c_3\bar{b}_3) - (d_1\bar{b}_1 + d_2\bar{b}_2 + d_3\bar{b}_3)$$

$$= \underbrace{(c_1 - d_1)}_{f_1}\bar{b}_1 + \underbrace{(c_2 - d_2)}_{f_2}\bar{b}_2 + \underbrace{(c_3 - d_3)}_{f_3}\bar{b}_3 = \vec{0}$$

$\{\bar{b}_1, \bar{b}_2, \bar{b}_3\}$ lin. uafh! $\Rightarrow f_1 = 0, f_2 = 0, f_3 = 0$

$c_1 - d_1 = 0$, $c_2 - d_2 = 0$, $c_3 - d_3 = 0$

$c_1 = d_1$, $c_2 = d_2$, $c_3 = d_3$

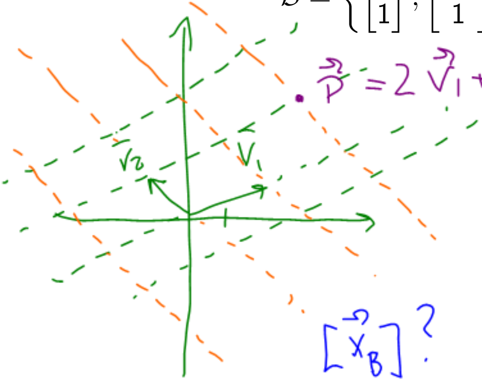
koordinaterne $\left[\vec{x}\right]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$ er

<u>entydigt</u> bestemt af $\vec{x}$.

Koordinatsystem eksempel

$$\begin{array}{ccc} & \bar{v}_1 & \bar{v}_2 \\ \mathcal{B} = \Big\{ & \begin{bmatrix} 2 \\ 1 \end{bmatrix}, & \begin{bmatrix} -1 \\ 1 \end{bmatrix} \Big\} \end{array} \qquad \vec{x} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \quad V = \operatorname{span}(\mathcal{B})$$

$\vec{p} = 2\vec{v}_1 + 1\vec{v}_2$



$[\vec{x}_{\mathcal{B}}]$?

$$[\vec{x}_{\mathcal{B}}] = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

metode 1

$$\vec{x} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} = d_1 \vec{v}_1 + d_2 \vec{v}_2 = d_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + d_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

ubekendte d_1, d_2, skal løse

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

totalmatrix

$$\left[\begin{array}{cc|c} 2 & -1 & 3 \\ 1 & 1 & 5 \end{array}\right] \overset{R_2 \leftrightarrow R_1}{\sim} \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 2 & -1 & 3 \end{array}\right]$$

$$\overset{R_2 \sim R_2 - 2R_1}{\sim} \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & -3 & -7 \end{array}\right]$$

$$R_2 \sim R_2/3 \quad \sim \begin{bmatrix} 1 & 1 & | & 5 \\ 0 & 1 & | & 7/3 \end{bmatrix}$$

$$R_1 = R_1 - R_2 \quad \sim \begin{bmatrix} 1 & 0 & | & 5 - \frac{7}{3} \\ 0 & 1 & | & \frac{7}{3} \end{bmatrix}$$

$$5 - \frac{7}{3} = \frac{15}{3} - \frac{7}{3} = \frac{8}{3}$$

$$\Rightarrow \quad d_1 = \frac{8}{3}, \quad d_2 = \frac{7}{3} \quad \text{eller} \quad [\vec{x}]_B = \begin{bmatrix} 8/3 \\ 7/3 \end{bmatrix}$$

metode 2

(virker, når $V = \mathbb{R}^n$)

skal finde

$$[\vec{x}]_{\mathcal{B}}: \quad \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{= \vec{x}} = \underbrace{\begin{bmatrix} \vec{v}_1 & \vec{v}_2 \end{bmatrix}}_{\mathcal{B}} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = d_1\vec{v}_1 + d_2\vec{v}_2$$

generelt: når $V = \mathrm{span}\{\bar{v}_1, \dots, \vec{v}_n\} = \mathbb{R}^n$

så er $B = [\bar{v}_1, \bar{v}_2, \dots, \bar{v}_n]$ invertérbar

$\Rightarrow B^{-1}$ findes!

så gjelder $[\vec{x}]_B = B^{-1}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$

(fordi $\vec{x} = B\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$

$$\Updownarrow \quad B^{-1}\vec{x} = \underbrace{B^{-1}B}_{I_n}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = I_n\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

$$= [\vec{x}]_B \;)$$