Opgavesæt 4 (SIF)

1.4#1

$$2x_1 + 6x_2 = -4$$

totalmatrix: $[2 \ 6 \mid -4]$

totalmatricen er i trappeform, der er ikke pivot i sidste søjle. $\Rightarrow$ ligningssystemet er konsistent

$$[2 \ 6 \mid -4] \overset{R_1 \to \frac{1}{2}R_1}{\sim} [1 \ 3 \mid -2] = [R \ \vec{c}]$$

(R er på reduceret trappeform)

i) ikke-pivot søjler i $R = [1 \ 3]$: søjle 2

$\Rightarrow$ x_2 er fri variabel

ii) ligningssystemet:

$$x_1 + 3x_2 = -2$$

$\Updownarrow$

$$x_1 = -2 - 3x_2 \qquad (x_2 \text{ fri})$$

iii) Indfør parameter s:

$$x_2 = s$$

iv) Generel løsning:

$$\begin{cases} x_1 = -2 - 3s & = -2 + (-3)s \\ x_2 = s & = 0 + 1 \cdot s \end{cases}, \quad s \in \mathbb{R}$$

v) vektorform:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} + s\begin{bmatrix} -3 \\ 1 \end{bmatrix}, \quad s \in \mathbb{R}.$$

1.4#5

$$\begin{cases} 2x_1 - 2x_2 + 4x_3 = 1 \\ -4x_1 + 4x_2 - 8x_3 = -3 \end{cases}$$

totalmatrix: $\left[\begin{array}{ccc|c} 2 & -2 & 4 & 1 \\ -4 & 4 & -8 & -3 \end{array}\right] \overset{R_2 \to R_2 + 2R_1}{\sim} \left[\begin{array}{ccc|c} 2 & -2 & 4 & 1 \\ 0 & 0 & 0 & -1 \end{array}\right]$

Totalmatricen har pivot-søjle i sidste søjle

$\Rightarrow$ ligningssystemet er ikke-konsistent!

1.4#9

$$\begin{cases} x_1 - x_2 - 3x_3 + x_4 = 0 \\ -2x_1 + x_2 + 5x_3 = -4 \\ 4x_1 - 2x_2 - 10x_3 + x_4 = 5 \end{cases}$$

totalmatrix:

$$\left[\begin{array}{cccc|c} 1 & -1 & -3 & 1 & 0 \\ -2 & 1 & 5 & 0 & -4 \\ 4 & -2 & -10 & 1 & 5 \end{array}\right] \underset{\sim}{R_2 \to R_2 + 2R_1} \left[\begin{array}{cccc|c} 1 & -1 & -3 & 1 & 0 \\ 0 & -1 & -1 & 2 & -4 \\ 4 & -2 & -10 & 1 & 5 \end{array}\right]$$

$R_3 \to R_3 - 4R_1$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & -3 & 1 & 0 \\ 0 & -1 & -1 & 2 & -4 \\ 0 & 2 & 2 & -3 & 5 \end{array}\right] \underset{\sim}{R_2 \to -R_2} \left[\begin{array}{cccc|c} 1 & -1 & -3 & 1 & 0 \\ 0 & 1 & 1 & -2 & 4 \\ 0 & 2 & 2 & -3 & 5 \end{array}\right]$$

$R_3 = R_3 - 2R_2$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & -3 & 1 & 0 \\ 0 & 1 & 1 & -2 & 4 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right] \quad \text{(trappeform)}$$

Den sidste søjle i totamatricen er ikke en pivot-søjle

= ligningssystemet er konsistent.

$$\left[\begin{array}{cccc|c} 1 & -1 & -3 & 1 & 0 \\ 0 & 1 & 1 & -2 & 4 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right] \underset{\sim}{R_2 \to R_2 + 2R_3} \left[\begin{array}{cccc|c} 1 & -1 & -3 & 1 & 0 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$$

$$\underset{\sim}{R_1 \to R_1 - R_3} \left[\begin{array}{cccc|c} 1 & -1 & -3 & 0 & 3 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right] \underset{\sim}{R_1 \to R_1 + R_2} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$$

i) Ikke-pivot søjler i

$$R = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad : \quad \text{søjle } 3$$

$\Rightarrow x_3$ er fri variabel

ii) $\begin{cases} 1\cdot x_1 + 0\cdot x_2 - 2x_3 + 0\cdot x_4 = 1 \\ 0\cdot x_1 + x_2 + x_3 + 0\cdot x_4 = -2 \\ 0\cdot x_1 + 0\cdot x_2 + 0\cdot x_3 + x_4 = -3 \end{cases}$

$\Updownarrow$

$$\begin{aligned} x_1 &= 1 + 2x_3 && = 1 + 2x_3 \\ x_2 &= -2 - x_3 && = -2 + (-1)\cdot x_3 \qquad (x_3 \text{ fri}) \\ x_4 &= -3 && = -3 + 0\cdot x_3 \end{aligned}$$

iii) Indfør parameter s

$x_3 = s$

iv) Generel løsning:

$$\begin{cases} x_1 = 1 + 2s \\ x_2 = -2 + (-1)s \\ x_3 = 0 + 1\cdot s \\ x_4 = -3 + 0\cdot s \end{cases} , \quad s \in \mathbb{R}$$

v) på vektorform:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \\ -3 \end{bmatrix} + s\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} , \quad s \in \mathbb{R}.$$

1.4#3

$$\begin{cases} x_1 - 2x_2 = -6 \\ -2x_1 + 3x_2 = 7 \end{cases} \qquad \text{totalmatrix: } \left[\begin{array}{cc|c} 1 & -2 & -6 \\ -2 & 3 & 7 \end{array}\right] = [A \mid \vec{b}]$$

$$\left[\begin{array}{cc|c} 1 & -2 & -6 \\ -2 & 3 & 7 \end{array}\right] \overset{R_2 \to R_2 + 2R_1}{\sim} \left[\begin{array}{cc|c} 1 & -2 & -6 \\ 0 & -1 & -5 \end{array}\right]$$

Sidste søjle i totalmatricen er ikke pivot-søjle

$\Rightarrow$ ligningssystemet er konsistent.

$$\left[\begin{array}{cc|c} 1 & -2 & -6 \\ 0 & -1 & -5 \end{array}\right] \overset{R_2 \to -R_2}{\sim} \left[\begin{array}{cc|c} 1 & -2 & -6 \\ 0 & 1 & 5 \end{array}\right] \overset{R_1 \to R_1 + 2R_2}{\sim} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 5 \end{array}\right]$$

(reduceret trappeform)

Der er ingen ikke-pivot søjler i $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, så der er ingen frie variable.

$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 5 \end{array}\right]$ svarer til ligningsystmet

$$\begin{cases} 1 \cdot x_1 + 0 \cdot x_2 = 4 \\ 0 \cdot x_1 + 1 \cdot x_2 = 5 \end{cases} \Rightarrow \begin{cases} x_1 = 4 \\ x_2 = 5 \end{cases}$$

Løsning på vektorform: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

1.4#7

$$\begin{cases} x_1 - 2x_2 - x_3 = -3 \\ 2x_1 - 4x_2 + 2x_3 = 2 \end{cases} \qquad \text{totalmatrix: } [A \mid \vec{b}] = \left[\begin{array}{ccc|c} 1 & -2 & -1 & -3 \\ 2 & -4 & 2 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -3 \\ 2 & -4 & 2 & 2 \end{array}\right] \overset{R_2 = R_2 - 2R_1}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & -1 & -3 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

sidste søjle i totalmatricen er ikke pivot-søjle

$\Rightarrow$ systemet er konsistent.

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -3 \\ 0 & 0 & 4 & 8 \end{array}\right] \underset{\sim}{R_2 \to \tfrac{1}{4}R_2} \left[\begin{array}{ccc|c} 1 & -2 & -1 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\underset{\sim}{R_1 \to R_1 + R_2} \left[\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] = [R \mid \vec{c}\,]$$

↖ reduceret trappeform.

i) ikke-pivot søjler i $R = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$: søjle 2

$\Rightarrow x_2$ er fri variabel

ii)
$$\begin{cases} 1 \cdot x_1 - 2x_2 + 0 \cdot x_3 = -1 \\ 0 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 = 2 \end{cases}$$

$$\Updownarrow$$

$$\begin{cases} x_1 = -1 + 2x_2 \\ x_3 = 2 \end{cases} \qquad (x_2 \text{ fri})$$

iii) indfør parameter s

$x_2 = s$

iv) Generel løsning:

$$\begin{aligned} x_1 &= -1 + 2s &&= -1 + 2 \cdot s \\ x_2 &= s &&= 0 + 1 \cdot s \\ x_3 &= 2 &&= 2 + 0 \cdot s \end{aligned} \qquad , s \in \mathbb{R}$$

v) på vektorform:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad s \in \mathbb{R}$$

1.4 #11

$$\begin{cases} x_1 + 3x_2 + x_3 + x_4 = -1 \\ -2x_1 - 6x_2 - x_3 = 5 \\ x_1 + 3x_2 + 2x_3 + 3x_4 = 2 \end{cases}$$

totalmatrix: $\left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & -1 \\ -2 & -6 & -1 & 0 & 5 \\ 1 & 3 & 2 & 3 & 2 \end{array}\right]$

$R_2 \to R_2 + 2R_1$ $\sim$ $R_3 \to R_3 - R_1$ $\left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & -1 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \end{array}\right]$

$R_3 \to R_3 - R_2$ $\sim$ $\left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & -1 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ (trappeform)

Sidste søjle i totalmatricen er ikke en pivot-søjle: ligningssystemet er konsistent.

$$\left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & -1 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 = R_2 - R_1$ $\sim$ $\left[\begin{array}{cccc|c} 1 & 3 & 0 & -1 & -4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ (reduceret trappeform)

i) ikke-pivot søjler i $R = \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$: Søjlerne 2 og 4

$\Rightarrow$ x_2 og x_4 er frie variable.

ii) ligningssystemet:

$$\begin{cases} 1 \cdot x_1 + 3x_2 + 0x_3 - 1 \cdot x_4 = -4 \\ 0 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 + 2x_4 = 3 \\ 0 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 + 0 \cdot x_4 = 0 \end{cases}$$

$$\Updownarrow \begin{cases} x_1 = -4 - 3x_2 + x_4 \\ x_3 = 3 - 2x_4 \end{cases} \quad (x_2, x_4 \text{ frie})$$

iii) indfør parametrene s, t

$x_2 = s$
$x_4 = t$

iv) Generel løsning:

$$\begin{cases} x_1 = -4 - 3s + t & = -4 + (-3)s + 1 \cdot t \\ x_2 = s & = 0 + 1 \cdot s + 0 \cdot t \\ x_3 = 3 - 2t & = 3 + 0 \cdot s + (-2)t \\ x_4 = t & = 0 + 0 \cdot s + 1 \cdot t \end{cases} \quad , \; s,t \in \mathbb{R}$$

v) <u>vektorform:</u>

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 3 \\ 0 \end{bmatrix} + s \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix} , \quad s,t \in \mathbb{R}.$$

1.4 # 17

totalmatrix:

$$\left[\begin{array}{cc|c} -1 & 4 & 3 \\ 3 & r & 2 \end{array}\right] \underset{\sim}{R_2 \to R_2 + 3R_1} \left[\begin{array}{cc|c} -1 & 4 & 3 \\ 0 & r+12 & 11 \end{array}\right]$$

hvis $r = -12$ er sidste søjle pivot-søjle

$\Rightarrow$ hvis $r = -12$ er systemet ikke-konsistent

1.4 # 19

totalmatrix:

$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 4 & -8 & r \end{array}\right] \underset{\sim}{R_2 = R_2 - 4R_1} \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & r \end{array}\right]$$

$\Rightarrow$ hvis $r \neq 0$ er sidste søjle pivot-søjle.

hvis $r \neq 0$ er systemet ikke-konsistent

1.4#21

totalmatrix:

$$\left[\begin{array}{cc|c} 1 & -3 & -2 \\ 2 & r & -4 \end{array}\right] \overset{R_2 \to R_2 - 2R_1}{\sim} \left[\begin{array}{cc|c} 1 & -3 & -2 \\ 0 & r+6 & 0 \end{array}\right]$$

sidste søjler er aldrig pivot-søjle.
systemet er konsistent for alle r.

1.4#37

$$A = \begin{bmatrix} -2 & 2 & 1 & 1 & -2 \\ 1 & -1 & -1 & -3 & 3 \\ -1 & 1 & -1 & -7 & 5 \end{bmatrix} \overset{R_1 \leftrightarrow R_2}{\sim} \begin{bmatrix} 1 & -1 & -1 & -3 & 3 \\ -2 & 2 & 1 & 1 & -2 \\ -1 & 1 & -1 & -7 & 5 \end{bmatrix}$$

$$\begin{array}{l} R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 + R_1 \\ \sim \end{array} \begin{bmatrix} 1 & -1 & -1 & -3 & 3 \\ 0 & 0 & -1 & -5 & 4 \\ 0 & 0 & -2 & -10 & 8 \end{bmatrix} \quad (3 \times 5\text{-matrix})$$

$$\overset{R_3 \to R_3 - 2R_2}{\sim} \begin{bmatrix} 1 & -1 & -1 & -3 & 3 \\ 0 & 0 & -1 & -5 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{trappeform})$$

2 pivot-søjler (søjle nr. 1 og søjle nr. 3)
3 ikke-pivot søjler.

rank(A) = 2
nullity(A) = 5 - 2 = 3

1.4#35

$$A = \begin{bmatrix} 1 & -1 & -1 & 0 \\ 2 & -1 & -2 & 1 \\ 1 & -2 & -2 & 2 \\ -4 & 2 & 3 & 1 \\ 1 & -1 & -2 & 3 \end{bmatrix} \begin{array}{c} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \\ \sim \\ R_4 \to R_4 + 4R_1 \\ R_5 \to R_5 - R_1 \end{array} \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & -1 & 2 \\ 0 & -2 & -1 & 1 \\ 0 & 0 & -1 & 3 \end{bmatrix}$$

$$\sim \begin{array}{l} R_3 \to R_3 + R_2 \\ R_4 \to R_4 + 2R_2 \end{array} \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & -1 & 3 \end{bmatrix} \begin{array}{c} R_4 \to R_4 - R_3 \\ R_5 \to R_5 - R_3 \\ \sim \end{array} \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3 pivot-søjler (søjle #: 1, 2, 3)
1 ikke-pivot søjle.

rank(A) = 3
nullity(A) = 4 - 3 = 1.

1.6#1

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 0 & 1 & 1 & 4 \\ 1 & 1 & 3 & 7 \end{array}\right] \overset{R_3 \to R_3 - R_1}{\sim} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 0 & 1 & 1 & 4 \\ 0 & 2 & 2 & 8 \end{array}\right]$$

$$\overset{R_3 \to R_3 - 2R_2}{\sim} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ (konsistent)

$\begin{bmatrix} -1 \\ 4 \\ 7 \end{bmatrix}$ ligger i $\operatorname{span}\left(\left\{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}\right\}\right)$. (ja)

1.6#3

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 5 \\ 1 & 1 & 3 & 2 \end{array}\right] \overset{R_3 \to R_3 - R_1}{\sim} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 5 \\ 0 & 2 & 2 & 2 \end{array}\right]$$

$$\overset{R_3 \to R_3 - 2R_2}{\sim} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & -8 \end{array}\right]$$ (<u>ikke</u> konsistent)

$\begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix}$ ligger <u>ikke</u> i $\operatorname{span}\left(\left\{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}\right\}\right)$. (nej)

1.6#7

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 3 & -1 \end{array}\right] \overset{R_3 \to R_3 - R_1}{\sim} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & 2 & -2 \end{array}\right]$$

$$\overset{R_3 \to R_3 - 2R_2}{\sim} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & -4 \end{array}\right]$$ (<u>ikke</u> konsistent)

$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ ligger ikke i $\text{span}\left(\left\{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}\right\}\right)$. (nej)

1.6#17

$$\left[\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 3 & r \\ -1 & 2 & -1 \end{array}\right] \overset{R_3 \to R_3 + R_1}{\sim} \left[\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 3 & r \\ 0 & 1 & 1 \end{array}\right]$$

$$\overset{R_3 \to R_3 - \frac{1}{3}R_2}{\sim} \left[\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 3 & r \\ 0 & 0 & 1 - \frac{r}{3} \end{array}\right]$$

hvis $1 - \frac{r}{3} = 0$ er systemet konsistent

$\Rightarrow$ hvis $r = 3$, så ligger $\begin{bmatrix} 2 \\ r \\ -1 \end{bmatrix}$ i $\text{span}\left(\left\{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}\right\}\right)$,

ellers ikke.

1.6#19

$$\left[\begin{array}{cc|c} -1 & 1 & 2 \\ 2 & -1 & r \\ 2 & 0 & -8 \end{array}\right] \underset{R_3 \to R_3 + 2R_1}{\overset{R_2 \to R_2 - R_3}{\sim}} \left[\begin{array}{cc|c} -1 & 1 & 2 \\ 0 & -1 & r+8 \\ 0 & 2 & -4 \end{array}\right] \overset{R_2 \leftrightarrow R_3}{\sim} \left[\begin{array}{cc|c} -1 & 1 & 2 \\ 0 & 2 & -4 \\ 0 & -1 & r+8 \end{array}\right]$$

$$\overset{R_2 \to \frac{1}{2}R_2}{\sim} \left[\begin{array}{cc|c} -1 & 1 & 2 \\ 0 & 1 & -2 \\ 0 & -1 & r+8 \end{array}\right] \overset{R_3 \to R_3 + R_2}{\sim} \left[\begin{array}{cc|c} -1 & 1 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & r+6 \end{array}\right]$$

$\Rightarrow$ hvis $r = -6$, er systemet konsistent, ellers ikke.

1.6#31

Hvis $A = \begin{bmatrix} 1 & 0 & -3 \\ -1 & 0 & 3 \end{bmatrix}$, er $A\vec{x} = \vec{b}$ konsistent for alle $\vec{b}$?

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & b_1 \\ -1 & 0 & 3 & b_2 \end{array}\right] \overset{R_2 \to R_2 + R_1}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -3 & b_1 \\ 0 & 0 & 0 & b_1 + b_2 \end{array}\right]$$

Hvis $b_1 = -b_2$, er systemed konsistent, ellers ikke.

Så svaret er <u>nej</u>.

1.6#33

Hvis $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ -2 & 2 \end{bmatrix}$, er $A\vec{x} = \vec{b}$ så konsistent for alle $\vec{b} \in \mathbb{R}^3$?

$$\left[\begin{array}{cc|c} 1 & -1 & b_1 \\ 0 & 1 & b_2 \\ -2 & 2 & b_3 \end{array}\right] \overset{R_3 \to R_3 + 2R_1}{\sim} \left[\begin{array}{cc|c} 1 & -1 & b_1 \\ 0 & 1 & b_2 \\ 0 & 0 & b_1 + b_3 \end{array}\right]$$

Hvis $b_1 = -b_3$, så er systemet konsistent, ellers ikke.

Så svaret er <u>nej.</u>