Trigonometri

Trigonometriske formler

$$\cos^2 x + \sin^2 x = 1$$

$$\tan x \cot x = 1, \qquad x \neq p\tfrac{\pi}{2}, \quad p \in \mathbb{Z}$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}, \qquad x \pm y \neq \tfrac{\pi}{2} + p\pi, \quad p \in \mathbb{Z}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}, \qquad x \neq \tfrac{\pi}{4} + p\tfrac{\pi}{2}, \quad p \in \mathbb{Z}$$

$$\cos x = \pm\sqrt{\frac{1 + \cos 2x}{2}}, \qquad \begin{cases} + \text{ for } -\frac{\pi}{2} + p2\pi \leq x \leq \frac{\pi}{2} + p2\pi \\ - \text{ for } \frac{\pi}{2} + p2\pi \leq x \leq \frac{3\pi}{2} + p2\pi \end{cases}, \quad p \in \mathbb{Z}$$

$$\sin x = \pm\sqrt{\frac{1 - \cos 2x}{2}}, \qquad \begin{cases} + \text{ for } p2\pi \leq x \leq \pi + p2\pi \\ - \text{ for } -\pi + p2\pi \leq x \leq p2\pi \end{cases}, \quad p \in \mathbb{Z}$$

$$\tan x = \frac{\sin 2x}{1 + \cos 2x} = \frac{1 - \cos 2x}{\sin 2x}, \qquad x \neq \tfrac{\pi}{2} + p\pi, \quad p \in \mathbb{Z}$$

$$\cos x = \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}$$

$$\sin x = \frac{2\tan\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}$$

$$\tan x = \frac{2\tan\left(\frac{x}{2}\right)}{1 - \tan^2\left(\frac{x}{2}\right)}, \qquad x \neq \tfrac{\pi}{2} + p\pi, \quad p \in \mathbb{Z}$$

$$\cos x < \frac{\sin x}{x} < 1, \qquad 0 < |x| < \tfrac{\pi}{2}$$

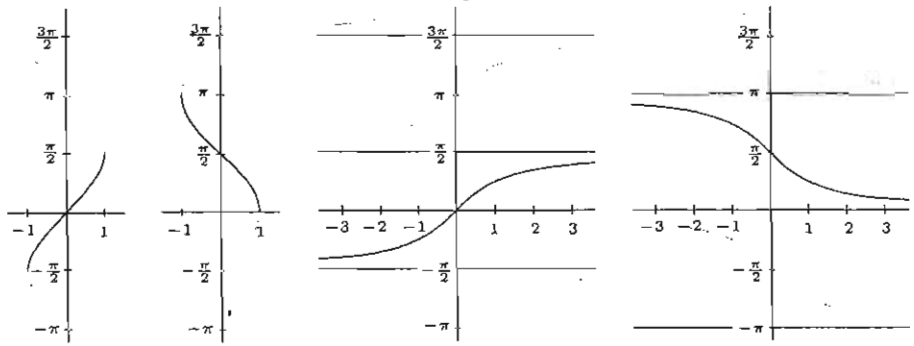
$$\sec x = \frac{1}{\cos x}, \qquad x \neq \tfrac{\pi}{2} + p\pi, \quad p \in \mathbb{Z}$$

$$\csc x = \frac{1}{\sin x}, \qquad x \neq p\pi, \quad p \in \mathbb{Z}$$

Eksakte værdier for udvalgte vinkler

Radian	sin	cos	tan	cot
0	0	1	0	∞
$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$\frac{5\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$
$\frac{\pi}{2}$	1	0	∞	0

Cirkulære funktioner (inverse trigonometriske funktioner)



$\arcsin x$ $\quad$ $\arccos x$ $\quad$ $\arctan x$ $\quad$ $\operatorname{arccot} x$

Trigonometriske grundligninger

$$\cos x = a, \quad |a| \leq 1 \quad \Leftrightarrow \quad x = \pm \arccos a + p2\pi, \qquad p \in \mathbb{Z}$$

$$\sin x = a, \quad |a| \leq 1 \quad \Leftrightarrow \quad x = \begin{cases} \arcsin a + p2\pi \\ \pi - \arcsin a + p2\pi \end{cases}, \qquad p \in \mathbb{Z}$$

$$\tan x = a \quad \Leftrightarrow \quad x = \arctan a + p\pi, \qquad p \in \mathbb{Z}$$

$A\cos x + B\sin x = C, \qquad 0 \leq |C| \leq \sqrt{A^2 + B^2}$

$\Leftrightarrow \cos(x - \varphi) = \frac{C}{\sqrt{A^2+B^2}}$,

hvor φ er valgt så $(\cos\varphi, \sin\varphi) = \left(\frac{A}{\sqrt{A^2+B^2}}, \frac{B}{\sqrt{A^2+B^2}}\right)$