On Success Probability in Random Network Coding
(A review)

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COST action meeting, Zurich, June 2013

...there is

random network coding

...and then there is

random network coding.

This talk is mostly on random network coding and only a little on random network coding.

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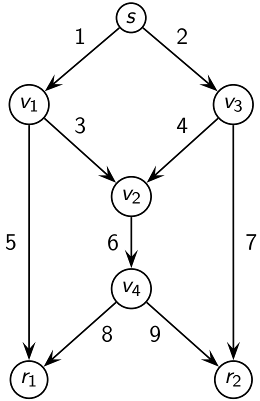
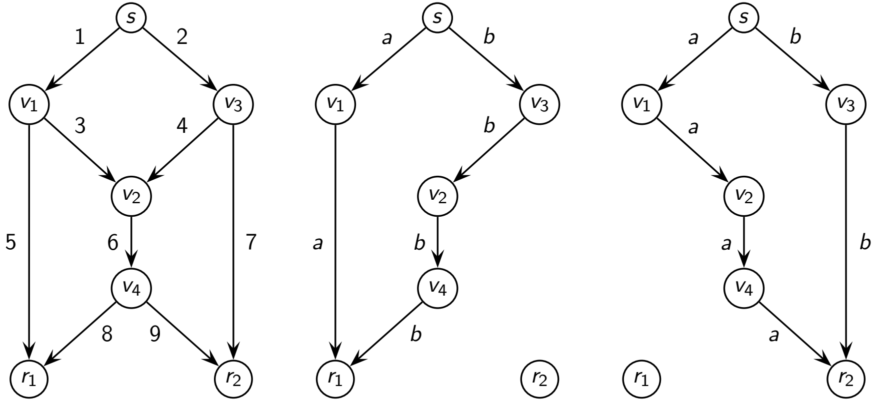
- Ho, T., Médard, M., Koetter, R., Karger, D.R., Effros, M., Shi, J., Leong, B.: *A Random Linear Network Coding Approach to Multicast*. IEEE Transactions on Information Theory, vol. 52, issue 10, pp. 4413–4430, October 2006.
- From description of COST Action IC 1104: "Random network coding emerged through an award-winning paper by R. Koetter and F. Kschischang in 2008 and has since then opened a major research area in communication technology with widespread applications for communication networks like ..."

Simplest possible network coding problem



Sender s wants to send two messages $a, b \in \mathbb{F}_2$ to both receivers r_1 and r_2 simultaneously.

Two partial solutions



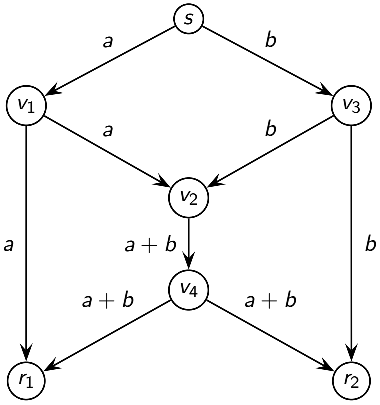
The network   Flow F_1 Flow F_2

The flow system is $\mathcal{F} = \{F_1, F_2\}$

$F_1 = \{(1,5),(2,4,6,8)\}, F_2 = \{(1,3,6,9),(2,7)\}$

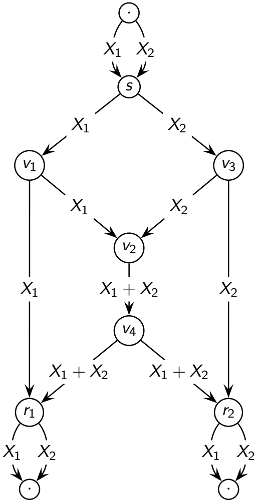
A solution

Routing is insufficient, but problem is solvable



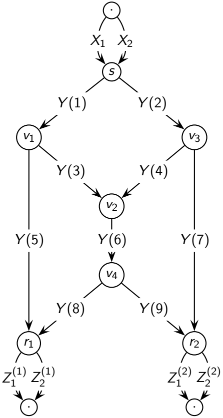
Receiver r_1 can reconstruct b as $a + (a + b)$
Receiver r_2 can reconstruct a as $(a + b) + b$

Linear network coding



Think of X_1 and X_2 as variables that take values in $\mathbb{F}_q = \mathbb{F}_2$.

Linear network coding



$$Y(1) = a_{11}X_1 + a_{21}X_2 = 1 \cdot X_1 + 0 \cdot X_2 = X_1$$
$$Y(2) = a_{12}X_1 + a_{22}X_2 = 0 \cdot X_1 + 1 \cdot X_2 = X_2$$

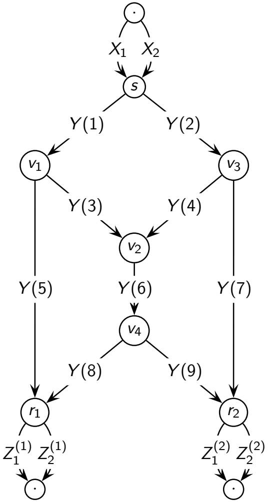
$$Y(3) = f_{13}Y(1) = 1 \cdot Y(1) = X_1$$
$$Y(4) = f_{24}Y(2) = 1 \cdot Y(2) = X_2$$
$$Y(5) = f_{15}Y(1) = 1 \cdot Y(1) = X_1$$
$$Y(6) = f_{36}Y(3) + f_{46}Y(4)$$
$$= 1 \cdot Y(3) + 1 \cdot Y(4) = X_1 + X_2$$

$$\vdots$$

$$Z_2^{(1)} = b_{52}^{(r_1)}Y(5) + b_{82}^{(r_1)}Y(8)$$
$$= 1 \cdot Y(5) + 1 \cdot Y(8) = X_1 + X_1 + X_2 = X_2$$

$$\vdots$$

Linear network coding

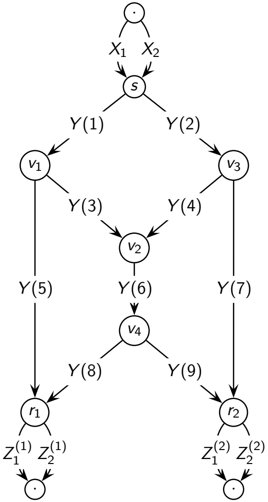


Alphabet is $\mathbb{F}_q = \mathbb{F}_2$ and coefficients below belong to $\mathbb{F}_q = \mathbb{F}_2$.

$$Y(j) = \sum_{i \in \text{in}(j)} f_{i,j} Y(i) + \sum_{K(X_i) = \text{tail}(j)} a_{i,j} X_i$$

$$Z_j^{(r_l)} = \sum_{i \in \text{in}(r_l)} b_{i,j}^{(r_l)} Y(i)$$

The matrix A



$$h = \#\{X_1, X_2\} = 2.$$

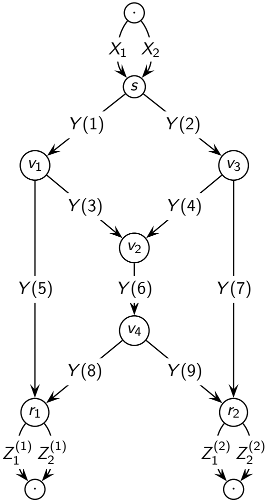
$$|E| = 9.$$

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A is $h \times |E|$.

The matrix F



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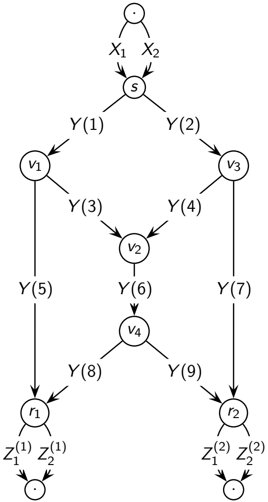
$$F = \begin{bmatrix} 0 & 0 & f_{13} & 0 & f_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_{24} & 0 & 0 & f_{27} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_{36} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_{46} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_{68} & f_{69} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

F is $|E| \times |E|$.

The matrix $B^{(r_i)}$



$h = \#\{X_1, X_2\} = 2$, $|E| = 9$.

$$B^{(r_1)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ b_{51}^{(r_1)} & b_{52}^{(r_1)} \\ 0 & 0 \\ 0 & 0 \\ b_{81}^{(r_1)} & b_{82}^{(r_1)} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$B^{(r_i)}$ is $|E| \times h$.

Matrices

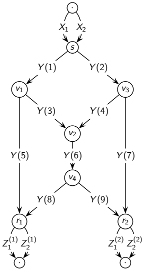
A is $h \times |E|$
$A_{i,j} = a_{i,j}$ if $K(X_i) = \text{tail}(j)$
$A_{i,j} = 0$ else

F is $|E| \times |E|$
$F_{i,j} = f_{i,j}$ if $i \in \text{in}(j)$
$F_{i,j} = 0$ else

For $l = 1, \ldots, |R|$

$B^{(r_l)}$ is $|E| \times h$
$B^{(r_l)}_{i,j} = b^{(r_l)}_{i,j}$ if $i \in \text{in}(r_l)$
$B^{(r_l)}_{i,j} = 0$ else

Paths in the network



$$F^2 =$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & (f_{13}f_{36}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (f_{24}f_{46}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & (f_{36}f_{68}) & (f_{36}f_{69}) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & (f_{46}f_{68}) & (f_{46}f_{69}) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Topological meaning of F^s

The $F_{i,j}$ "holds" information on all paths of length 2 starting in edge i and ending in edge j.

The (i,j)th entry of F^n "holds" information on all paths of length $n+1$ starting in edge i and ending in edge j.

$$\left(F^n\right)_{i,j} = \sum_{\substack{(i=j_0,j_1,\ldots,j_n=j) \\ \text{a path} \\ \text{in } G}} f_{i=j_0,j_1} f_{j_1,j_2} \cdots f_{j_{n-1},j_n=j}$$

G being cycle free $F^N = 0$ for some big enough N.

$I + F + \cdots + F^{N-1}$ holds information on all paths of any length.

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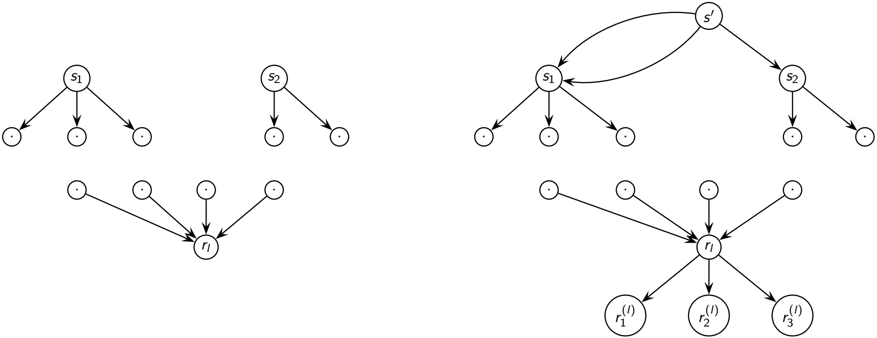

Modification of network. In original network two sources at s_1 and one source at s_2.

In modified network the $a_{i,j}$'s and the $b_{i,j}^{(r_l)}$'s from the original network play the same role as the $f_{i,j}$'s

Transfer matrix

Lemma:
$M^{(r_l)} = A(I + F + \cdots + F^{N-1})B^{(r_l)}$
holds information on all paths from s' to $\{r_1^{(l)}, \ldots, r_h^{(l)}\}$

From this we derive:

Theorem: $(X_1, \ldots, X_h)M^{(r_l)} = (Z_1^{(r_l)}, \ldots Z_h^{(r_l)})$

$M^{(r_l)}$ is called the transfer matrix for r_l

Transfer polynomial

For successful encoding/decoding we require
$M^{(r_1)} = \cdots = M^{(r_{|R|})} = I$

Relaxed requirement:
$\det(M^{(r_l)}) \neq 0$ for $l = 1, \ldots, |R|$.

Success iff
$\prod_{l=1,\ldots,|R|} \det(M^{(r_l)}) \neq 0$

Considered as a polynomial in the $a_{i,j}$'s, $f_{i,j}$'s and $b_{i,j}^{(r_l)}$'s this product is called the transfer polynomial.

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Topological meaning of det M^{r_l}

Theorem: The permanent per$(M^{(r_l)})$ is the sum of all monomial expressions in the $a_{i,j}$'s, $f_{i,j}$'s and $b_{i,j}^{(r_l)}$'s which correspond to a flow of size h from s' to $\{r_1^{(l)}, \ldots, r_h^{(l)}\}$ in the modified graph.

Proof: Apply the lemma carefully.

As a consequence det$(M^{(r_l)})$ is a linear combination of the expressions corresponding to flows. The coefficients being 1 or -1.

In the transfer polynomial $\prod_{l=1,\ldots,|R|} \det(M^{(r_l)})$ every monomial corresponds to a flow system.

Coefficients are integers
which in $\mathbb{F}_q$ becomes elements in $\mathbb{F}_p$, p being the characteristic.

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Main theorem on linear network coding

Terms MAY cancel out when taking the product of the $\det(M^{(r_l)})$'s.

If all $\det(M^{(r_l)})$'s are different from 0 then so is the transfer polynomial.

Theorem: A multicast problem is solvable iff the graph contains a flow system of size h. If solvable then solvable with linear network coding whenever $q \geq |R|$.

Proof (almost): Necessity follows from unicast considerations. Assume a flow system exists. The transfer polynomial is non-zero and no indeterminate appears in power exceeding $|R|$. Therefore if $q > |R|$ then over $\mathbb{F}_q$ a non-zero solution exists according to the footprint bound.

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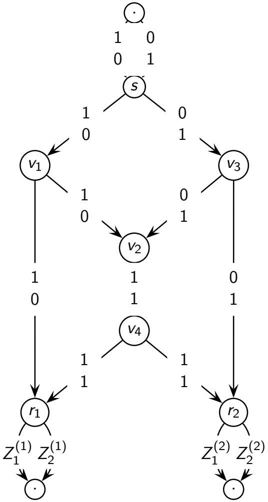
Global coding vectors



...actually these vectors later inspired Kötter et. al. to consider the subspace codes

Global coding vectors

In linear network coding we always have
$Y(i) = c_1 X_1 + \cdots + c_h X_h$ for some $c_1, \ldots, c_h \in \mathbb{F}_q$.

We shall call $(c_1, \ldots, c_h)$ the global coding vector for edge i.

A receiver that does not know how encoding was done can learn how to decode (if possible) as follows.

Senders inject into the system h message vectors
$(1, 0, \cdots, 0), (0, 1, 0 \ldots, 0), \ldots, (0, \ldots, 0, 1)$.

These generate the global coding vectors at each edge including the in edges of r_l.

If the received global coding vectors span $\mathbb{F}_q^h$ then proper $b_{i,j}^{(r_l)}$'s can be found.

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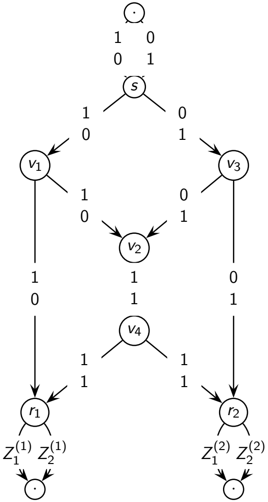
Jaggi-Sanders algorithm



- Start by localizing a flow system.
- At the top of the flow system choose a basis.
- Update local coding coefficients edge by edge moving down the flow system, in a way such that basis is kept in the cut.

Jaggi-Sanders algorithm

Jaggi-Sanders algorithm takes as input a solvable multicast problem.
It adds a new source s' and moves all processes to this point and add edges $e_1, \dots, e_h$ from s' to S.
In the extended graph a flow system is found.

The algorithm for every receiver keeps a list of edges corresponding to a cut.

Also it updates along the way encoding coefficients in such a way that the global coding vectors corresponding to any of the $|R|$ cuts at any time span the whole of $\mathbb{F}_q^h$.

Edges in the flow system are visited according to an ancestral ordering.

In every update at most one edge is replaced in a given cut.

The Jaggi-Sanders algorithm cont.

Lemma 1.1: Given a basis $\{\vec{b}_1, \ldots, \vec{b}_h\}$ for $\mathbb{F}_q^h$
and $\vec{c} \in \mathbb{F}_q^h$,
there is exactly one choice of $a \in \mathbb{F}_q$ such that
$\vec{c} + a\vec{b}_h \in \text{span}_{\mathbb{F}_q}\{\vec{b}_1, \ldots, \vec{b}_{h-1}\}$.

From the Jaggi-Sanders algorithm we get $q \geq |R|$ is enough!!!
(the zero-solution does not work for any receiver)

Random network coding

In random network coding a (possibly empty) subset of the $a_{i,j}{}'s, f_{i,j}{}'s$ are chosen *a priori* in such a way that the resulting network coding problem is still solvable.

Remaining encoding coefficients are chosen in a distributed manner.
They are chosen independently by uniform distribution.

The transfer polynomial with the *a priori* chosen coefficients plugged in considered as a polynomial with coefficients in $\mathbb{F}_q(b_{i,j}^{(r)}{}'s)$, is called the *a priori* transfer polynomial.

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Success probability

Assume the *a priori* transfer polynomial F is non-zero.

Let $T_1^{i_1} \cdots T_m^{i_m}$ be its leading monomial with respect to $\prec$.
The number of combinations of $a_{i,j}{}'s$, $f_{i,j}{}'s$ that plugged into F give a non-zero element in $\mathbb{F}_q(b_{i,j}^{(r)}{}'s)$ is at least $(q-i_1)\cdots(q-i_m)$ (the footprint bound)

$\lim_{q\to\infty} \frac{(q-i_1)\cdots(q-i_m)}{q^m} = 1.$

Recall, $b_{i,j}^{(r_l)}$ appears in power at most 1.

For each of the above solutions:
$b_{i,j}^{(r_l)}$ can be chosen such that F evaluates to non-zero in $\mathbb{F}_q$.

In conclusion: $P_{\text{succ}} \geq (q-i_1)\cdots(q-i_m)/q^m = P_{\text{FP2}}$
(Matsumoto-Thomsen-G.)

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Success probability - cont.

Any monomial in the transfer polynomial corresponds to a flow system

$$
\begin{aligned}
P_{\text{succ}} &\geq \min\{(q-i_1)\cdots(q-i_m)/q^m \mid X_1^{i_1}\cdots X_m^{i_m} \text{ corresponds} \\
&\qquad\qquad\qquad\qquad \text{to a flow system in } G\} \\
&= P_{\text{FP1}}
\end{aligned}
$$

(Matsumoto-Thomsen-G.). Note

- not all flow systems need to appear in transfer polynomial
- not all monomials can be chosen as leading

Success probability - cont.

Lemma 1.2: Let $F \in k[T_1, \ldots, T_m] \backslash \{0\}$ where k is a field containing $\mathbb{F}_q$. Assume all monomials $T_1^{j_1} \cdots T_m^{j_m}$ in the support of F satisfies

1. $j_1, \ldots, j_m \leq d$, where d is some fixed number $d \leq q$.
2. $j_1 + \cdots + j_m \leq dN$ for some fixed integer N with $N \leq m$

The probability that F evaluates to a non-zero value when $(X_1, \ldots, X_m) \in \mathbb{F}_q^m$ is chosen by random (uniformly) and is plugged into F is at least

$$\left(\frac{q-d}{q}\right)^N$$

Proof 1: A lot of technical lemmas and the Schwartz-Zippel bound.

Proof 2: The footprint bound plus one simple observation.

Success probability - cont.

Every monomial in transfer polynomial comes from a flow system $\mathcal{F} = (F_1, \ldots, F_{|R|})$. Consider all possible flows (not systems).

For each flow count the number η of encoding coefficients (not chosen *a priori*). Let η' be the maximum of the numbers η. Then for all monomials we have cond. 1 and cond. 2 with $d = |R|$ and $N = \eta'$

We get

$$P_{\text{succ}} \geq \left(\frac{q - |R|}{q}\right)^{\eta'} = P_{\text{Ho2}}$$

Clearly $\eta' \leq |E|$ which gives

$$P_{\text{succ}} \geq \left(\frac{q - |R|}{q}\right)^{|E|} = P_{\text{Ho1}}$$

Success probability - cont.

Every monomial in transfer polynomial comes from a flow system $\mathcal{F} = (F_1, \ldots, F_{|R|})$. Consider all possible flows (not systems).

For each flow count the number η of encoding coefficients (not chosen *a priori*). Let η' be the maximum of the numbers η. Then for all monomials we have cond. 1 and cond. 2 with $d = |R|$ and $N = \eta'$

We get

$$P_{\text{succ}} \geq \left(\frac{q - |R|}{q}\right)^{\eta'} = P_{\text{Ho2}}$$

Clearly $\eta' \leq |E|$ which gives

$$P_{\text{succ}} \geq \left(\frac{q - |R|}{q}\right)^{|E|} = P_{\text{Ho1}}$$

Success probability - cont.

$$P_{\text{Ho1}} \leq P_{\text{Ho2}} \leq P_{\text{FP1}} \leq P_{\text{FP2}}$$

Applying the Jaggi-Sanders point of view one get "flow-bounds". These are always better than P_{Ho2}.

Combinatorial approach:

- Jaggi-Sanders visit edges in flowsystem one by one.
- Alternative approach by Balli, Yan and Zhang: Visit vertices in flowsystem one by one. Gives bound in terms of number of vertices.

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The operator channel as used in random network coding could be based on:

- Linear random network coding.
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