Transp 6.1

$$W \in \{1, 2, \ldots, w, \ldots, 2^{nR}\} \qquad \mathcal{C} = \begin{array}{ccc} x_1(1) & \cdots & x_n(1) \\ x_1(2) & \cdots & x_n(2) \\ \vdots & & \vdots \\ x_1(w) & \cdots & x_n(w) \\ \vdots & & \\ x_1(2^{nR}) & \cdots & x_n(2^{nR}) \end{array} \xrightarrow{\text{kanal}} P(y^n \mid x^n(w))$$

y^n dekodes som $\hat{w}$, netop hvis

$(x^n(\hat{w}), y^n) \in A_\varepsilon^{(n)}$

og

$(x^n(w'), y^n) \notin A_\varepsilon^{(n)}$ for $w' \neq \hat{w}$.

$P_e^{(n)}(\mathcal{C})$

$= \frac{1}{2^{nR}}(\lambda_1 + \lambda_2 + \cdots + \lambda_w + \cdots + \lambda_{2^{nR}})$

Transp 6.1

$$\mathcal{C}$$

$$W \in \{1, 2, \dots w, \dots 2^{nR}\} \qquad \begin{array}{ccc} X_1(1) & \dots & X_n(1) \\ X_1(2) & \dots & X_n(2) \\ \vdots & & \vdots \\ X_1(w) & \dots & X_n(w) \\ \vdots & & \\ X_1(2^{nR}) & \dots & X_n(2^{nR}) \end{array} \quad \xrightarrow{\text{kanal}} P(y^n \mid X^n(w))$$

y^n dekodes som $\hat{w}$, netop hvis

$$(x^n(\hat{w}), y^n) \in A_\varepsilon^{(n)}$$

og

$$(x^n(w'), y^n) \notin A_\varepsilon^{(n)} \quad \text{for } w' \neq \hat{w}.$$

$$P_e^{(n)}(\mathcal{C})$$

$$= \frac{1}{2^{nR}} (\lambda_1 + \lambda_2 + \dots + \lambda_w + \dots + \lambda_{2^{nR}})$$

Transp 6.2

$$\Pr(\mathcal{E}) = \langle P_e^{(n)}(\mathcal{C}) \rangle_{\mathcal{C}}$$

$$= \sum_{\mathcal{C}} P(\mathcal{C}) P_e^{(n)}(\mathcal{C})$$

$$= \sum_{\mathcal{C}} P(\mathcal{C}) \frac{1}{2^{nR}} \sum_{w=1}^{2^{nR}} \lambda_w(\mathcal{C})$$

$$= \frac{1}{2^{nR}} \sum_{w=1}^{2^{nR}} \sum_{\mathcal{C}} P(\mathcal{C}) \lambda_w(\mathcal{C})$$

$$= \sum_{\mathcal{C}} P(\mathcal{C}) \lambda_1(\mathcal{C})$$

$$= \langle \lambda_1(\mathcal{C}) \rangle_{\mathcal{C}} .$$

$$(x^n(1), x^n(2), \ldots, x^n(2^{nR}), y^n) \in \underbrace{\mathcal{X}^n \times \cdots \times \mathcal{X}^n}_{2^{nR}} \times \mathcal{Y}^n$$

$$\lambda_1(\mathcal{C}) = \sum_{\substack{y^n: \\ (x^n(1), \ldots, x^n(2^{nR}), y^n) \in E_1^c \cup E_2 \cup \ldots \cup E_{2^{nR}}}} p(y^n \mid x^n(1)) .$$

$E_i = \{(x^n(1), \ldots, x^n(2^{nR}), y^n) \mid (x^n(i), y^n)$ sammenhørende typisk$\}$.
$(\in A_\varepsilon^{(n)})$

$$\langle \lambda_1(\mathcal{C}) \rangle_{\mathcal{C}} = \sum_{\mathcal{C}} P(\mathcal{C}) \lambda_1(\mathcal{C})$$

$$= \sum_{\substack{x^n(1), \ldots, x^n(2^{nR}), y^n: \\ (x^n(1), \ldots, x^n(2^{nR}), y^n) \in E_1^c \cup E_2 \cup \ldots \cup E_{2^{nR}}}} P(x^n(1), \ldots, x^n(2^{nR}))\, P(y^n \mid x^n(1))$$

$$= P(E_1^c \cup E_2 \cup \ldots \cup E_{2^{nR}})$$

$$\leq P((\underline{X}^n(1), \underline{Y}^n) \notin E_1) + \sum_{i=2}^{2^{nR}} P((\underline{X}^n(i), \underline{Y}^n) \in E_i) .$$

$$
\begin{aligned}
\langle P_e^n(\mathcal{C}) \rangle &= \langle \lambda_1(\mathcal{C}) \rangle_{\mathcal{C}} \\
&\leq \varepsilon + \sum_{i=2}^{2^{nR}} 2^{-n(I(\underline{X};Y)-3\varepsilon)} \\
&\leq \varepsilon + 2^{nR} \cdot 2^{-n(I(\underline{X};Y)-3\varepsilon)}
\end{aligned}
$$

- $p(x) = p^*(x)$ vælges så $I(\underline{X};Y) = C$
- $\mathcal{C} = \mathcal{C}^*$ vælges så $P_e^n(\mathcal{C}^*) \leq \langle P_e^n(\mathcal{C}) \rangle$. (*)

Så gælder for $R < C - 3\varepsilon$ (**)

$$
\begin{aligned}
P_e^n(\mathcal{C}^*) &\overset{(*)}{\leq} \varepsilon + 2^{nR} \cdot 2^{-n(C-3\varepsilon)} \\
&\overset{(**)}{\leq} 2\varepsilon \text{ for tilstrækkeligt store } n.
\end{aligned}
$$

- Lad, efter evt. omnummerering,

$$\lambda_1(\mathcal{C}^*) \leq \lambda_2(\mathcal{C}^*) \leq \ldots \leq \lambda_{2^{nR}}(\mathcal{C}^*).$$

Så gælder for $\mathcal{C}^*_{udt.}$, som består af de første $\lceil \frac{1}{2} 2^{nR} \rceil$ kodeord i $\mathcal{C}^*$, at

$\lambda^{(n)}(\mathcal{C}^*_{udt}) \cancel{P_e^{(n)}(\mathcal{C}^*_{udt})} \leq 2 P_e^{(n)}(\mathcal{C}^*) \leq 4\varepsilon$ for tilstrækkeligt store n,

og $R(\mathcal{C}^*_{udt.}) = \frac{1}{n} \log \lceil \frac{1}{2} 2^{nR} \rceil \geq \frac{1}{n} \log(\frac{1}{2} 2^{nR}) = R - \frac{1}{n}$.

Transp 6.5

Lad $A = \frac{1}{N} \sum_{n=1}^{N} a_n$, hvor $0 \leq a_1 \leq a_2 \leq \cdots \leq a_N$.

Så gælder $a_{\lceil N/2 \rceil} \leq 2A$.

Bevis Der gælder

$$A = \frac{1}{N} \sum_{n=1}^{N} a_n$$

$$\geq \frac{1}{N} \sum_{n=\lceil N/2 \rceil}^{N} a_n$$

$$\geq \frac{1}{N} \left(N - \left(\lceil \tfrac{N}{2} \rceil - 1 \right) \right) a_{\lceil N/2 \rceil}$$

$$\geq \frac{1}{N} \cdot \frac{N}{2} \, a_{\lceil N/2 \rceil}$$

$$= \frac{1}{2} a_{\lceil N/2 \rceil} ,$$

hvoraf

$$a_{\lceil N/2 \rceil} \leq 2A \qquad \square$$