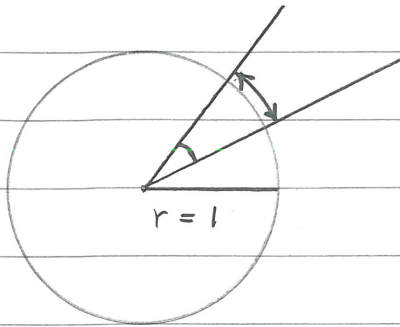


1. kursusgang: Trigonometriske funktioner og deres inverse

Radian målet: Vinkelmaal ved buelængde på enhedscirklen



Omkreds af enhedscirklen:

$$2\pi r = 2\pi \cdot 1 = 2\pi$$

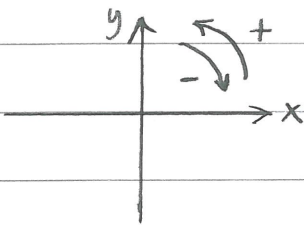
$$2\pi \text{ rad} = 360^\circ$$

$$\pi \text{ rad} = 180^\circ$$

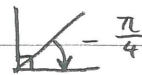
$$1 \text{ rad} = \frac{180^\circ}{\pi}$$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

Positive/negative vinkler

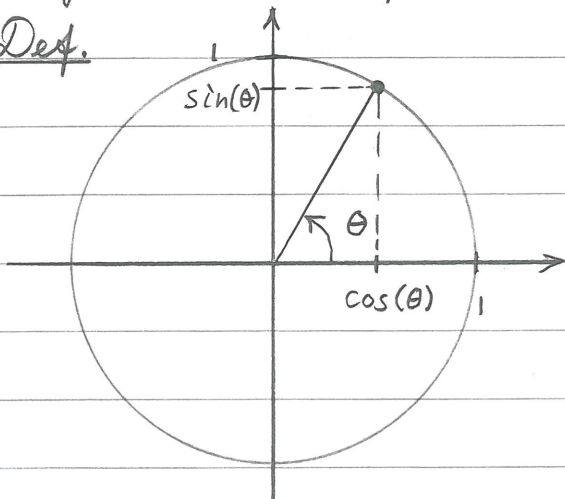


Eks. $-45^\circ = -45 \cdot \frac{\pi}{180} \text{ rad} = -\frac{\pi}{4} \text{ rad}$



Trigonometriske funktioner

Def.



$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

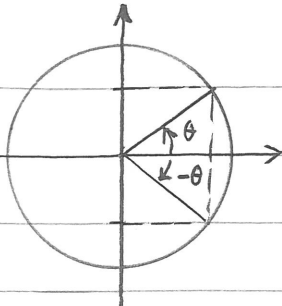
$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

Trigonometriske identiteter

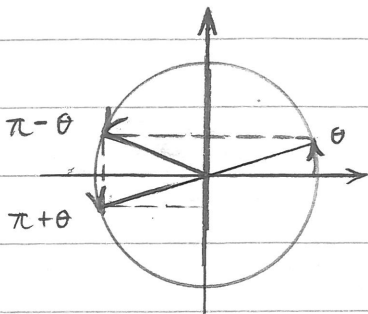
Da afstanden fra $(0,0)$ til $(\cos(\theta), \sin(\theta))$ er 1, har vi

$$\cos^2(\theta) + \sin^2(\theta) = 1$$



$$\cos(-\theta) = \cos(\theta)$$

$$\sin(-\theta) = -\sin(\theta)$$



$$\cos(\pi + \theta) = -\cos(\theta)$$

$$\cos(\pi - \theta) = -\cos(\theta)$$

$$\sin(\pi + \theta) = -\sin(\theta)$$

$$\sin(\pi - \theta) = \sin(\theta)$$

$$\cos(2\pi + \theta) = \cos(\theta) \quad , \quad \sin(2\pi + \theta) = \sin(\theta) .$$

De trigonometriske additionsformler

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) \quad ,$$

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) .$$

(Bevis udeladt)

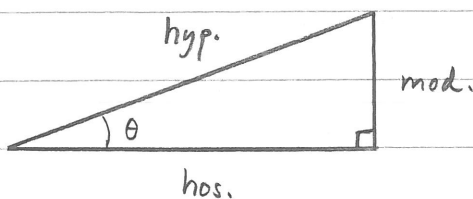
Sættes $\alpha = \beta = \theta$ fås dobbelt-vinkel formlerne

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 1 - 2\sin^2(\theta)$$

$$= 2\cos^2(\theta) - 1 \quad ,$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) .$$

Retvinklede trekanter



θ er en spids vinkel i en retvinklet trekant. Vi kan skalere med $\frac{1}{\text{hyp.}}$ og få følgende formler:

$$\cos(\theta) = \frac{\text{hos}}{\text{hyp}} \quad , \quad \sin(\theta) = \frac{\text{mod}}{\text{hyp}} \quad , \quad \tan(\theta) = \frac{\text{mod}}{\text{hos}} \quad ,$$

$$\sec(\theta) = \frac{\text{hyp}}{\text{hos}} \quad , \quad \csc(\theta) = \frac{\text{hyp}}{\text{mod}} \quad , \quad \cot(\theta) = \frac{\text{hos}}{\text{mod}}$$

Spezielle Winkel

θ	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin(\theta)$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos(\theta)$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$

