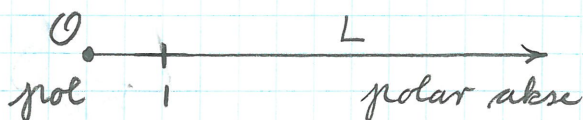


2. kursusgang : Polære koordinater

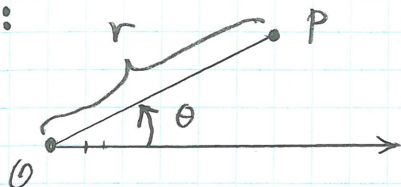
O : Fast reference punkt i planen (pol)

L : Fast halvakse startende i O (polar akse)

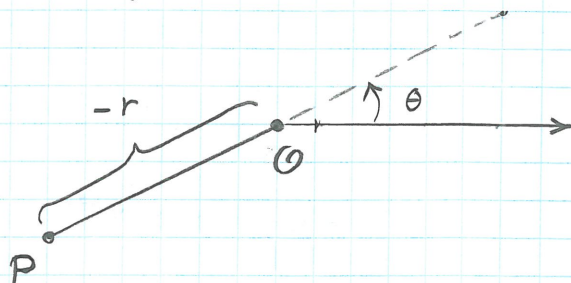


Def. Punktet P med polære koordinater (r, θ) :

$r > 0$:

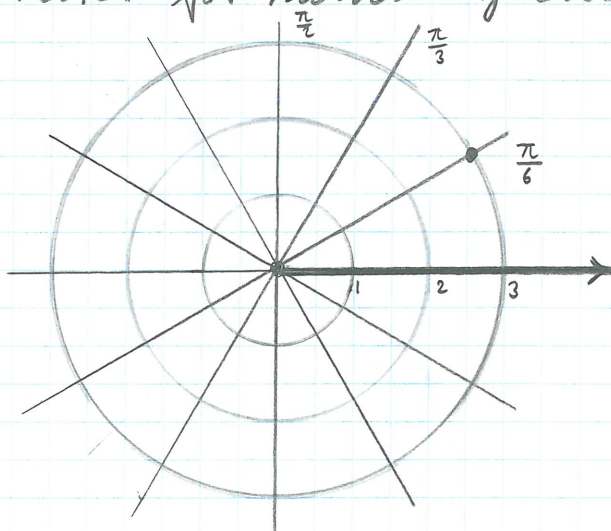


$r < 0$:

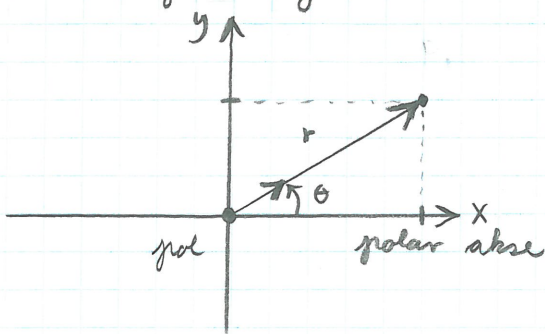


$r = 0$: $(0, \theta)$ er polære koordinater for O for ethvert θ .

Bemærk : $(r, \theta + 2n\pi)$ og $(-r, \theta + (2n+1)\pi)$, $n \in \mathbb{Z}$ er polære koordinater for samme punkt.



Omregning mellem polære og rektangulære koordinater



$$\begin{bmatrix} x \\ y \end{bmatrix} = r \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$$

polære \rightarrow rektangulære

$$x = r \cos \theta,$$

$$y = r \sin \theta.$$

rektangulære \rightarrow polære

$$r = \sqrt{x^2 + y^2}$$

$$\tan(\theta) = \frac{y}{x}, \quad x \neq 0.$$

Aegn!

①

$$\theta = \arctan\left(\frac{y}{x}\right), \quad x > 0$$

$$\theta = \arctan\left(\frac{y}{x}\right) + \pi, \quad x < 0$$

Øks: Omregn $(x, y) = (-1, \sqrt{3})$ til polære koordinater.

$$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2,$$

$$\theta = \arctan\left(\frac{\sqrt{3}}{-1}\right) + \pi = \arctan(-\sqrt{3}) + \pi = -\frac{\pi}{3} + \pi = \frac{2\pi}{3}.$$

$$(r, \theta) = \underline{\underline{\left(2, \frac{2\pi}{3}\right)}}$$

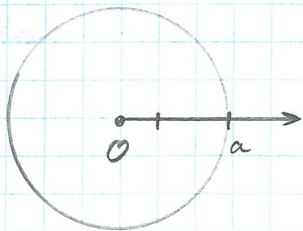
Polære koordinatligninger

$$F(r, \theta) = 0$$

Grafen for denne ligning er punktmængden

$$\{(r, \theta) \mid F(r, \theta) = 0\}.$$

Øks: $r = a$, hvor $a > 0$.



Circle med centrum i O og radius a .

(Ligning i rektangulære koordinater:
 $x^2 + y^2 = a^2$)

Øks: $r = 2a \cos(\theta)$, hvor $a > 0$.

$$r = 2a \cos(\theta) \Leftrightarrow$$

$$\left\{ r = 2a \cos(\theta) \wedge \theta \neq \frac{\pi}{2} + n\pi, n \in \mathbb{Z} \right\} \vee r = 0 \Leftrightarrow$$

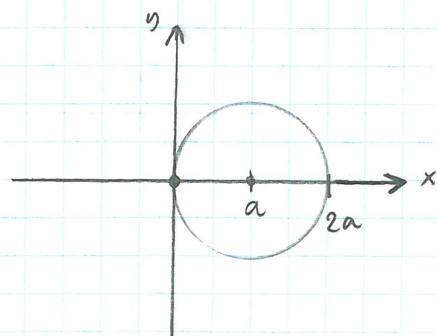
$$r^2 = 2a r \cos(\theta) \Leftrightarrow$$

$$x^2 + y^2 = 2a x \Leftrightarrow$$

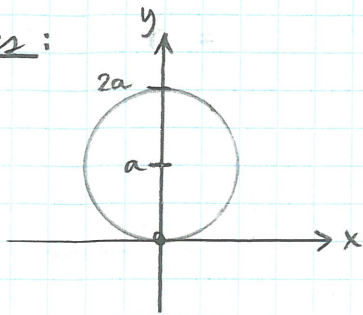
$$x^2 - 2ax + y^2 = 0 \Leftrightarrow$$

$$(x - a)^2 - a^2 + y^2 = 0 \Leftrightarrow$$

$$(x - a)^2 + y^2 = a^2$$

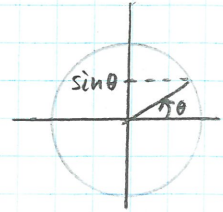


Eks:



$$r = 2a \sin(\theta) \quad , \quad \text{hvor } a > 0$$

Eks: $r = 2 + 2 \sin \theta$

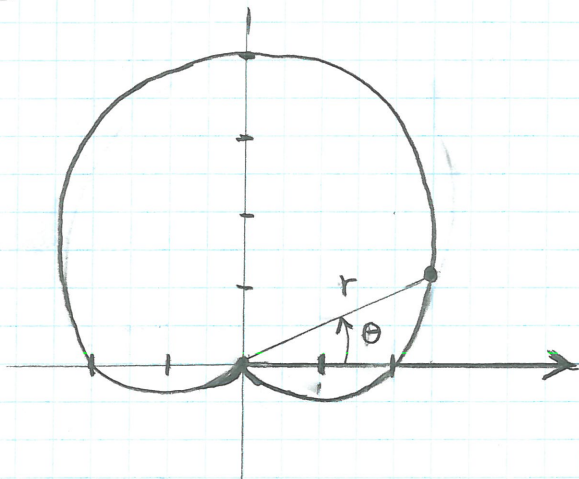


$0 < \theta < \frac{\pi}{2}$: r vokser fra 2 til 4

$\frac{\pi}{2} < \theta < \pi$: r aftager fra 4 til 2

$\pi < \theta < \frac{3\pi}{2}$: r aftager fra 2 til 0

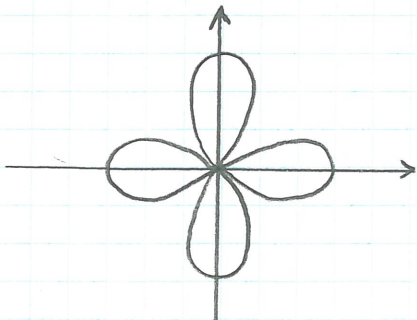
$\frac{3\pi}{2} < \theta < 2\pi$: r vokser fra 0 til 2



Cardioide

Cardioider : $r = a(1 \pm \sin \theta)$, $r = a(1 \pm \cos \theta)$,
hvor $a > 0$.

Eks: $r = a \cos(2\theta)$, $a > 0$



Firebladet rose