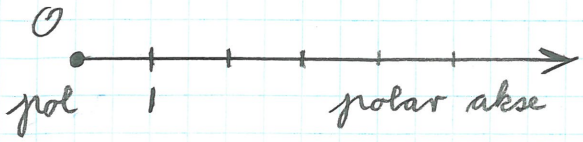
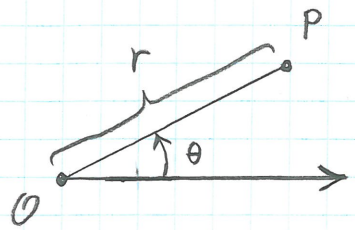
3. kursusgang : Repetition

Polære koordinater

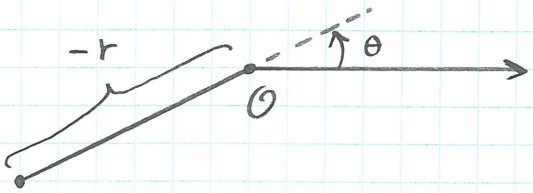


Def. Punktet P med polære koordinater (r, θ):

$r > 0$:

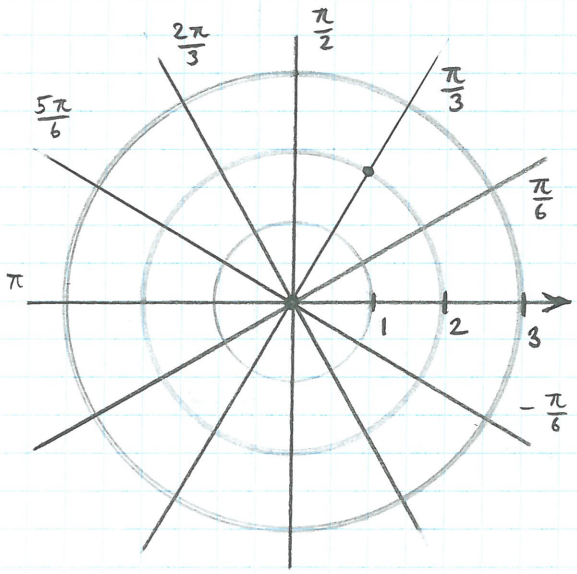


$r < 0$:

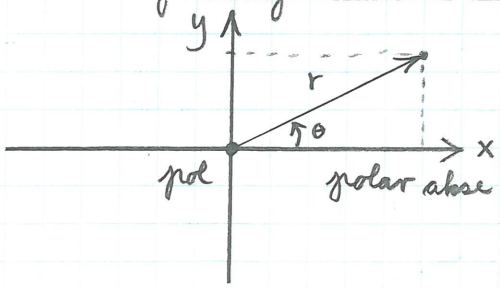


$r = 0$: $(0, \theta)$ er polære koordinater for polen O for ~~ethvert~~ θ.

Bemærk: $(r, \theta + 2\pi p)$ og $(-r, \theta + \pi + 2\pi p)$, $p \in \mathbb{Z}$ er polære koordinater for samme punkt.



Omregning mellem polære og rektangulære koordinater



$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$x^2 + y^2 = r^2\cos^2\theta + r^2\sin^2\theta = r^2(\cos^2\theta + \sin^2\theta) = r^2$$

$$x \neq 0: \quad \frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} = \tan\theta$$

polære ⇝ rektangulære

$$x = r\cos\theta$$

$$y = r\sin\theta$$

rektangulære ⇝ polære

$$r = \sqrt{x^2+y^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right), \quad x > 0$$

$$\theta = \arctan\left(\frac{y}{x}\right) + \pi, \quad x < 0$$

$$\theta = \frac{\pi}{2}, \quad x = 0, \ y > 0$$

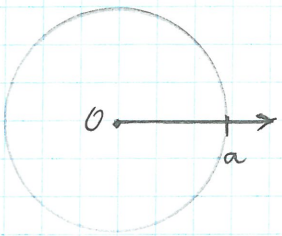
$$\theta = -\frac{\pi}{2}, \quad x = 0, \ y < 0$$

Polære koordinatligninger med tilhørende grafer

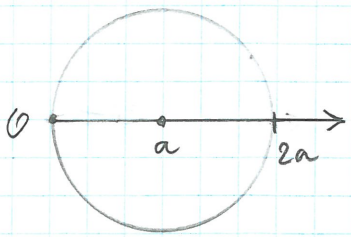
Ligning : $F(r,\theta) = 0$

Graf : $\{(r,\theta) \mid F(r,\theta) = 0\}$

Eks.

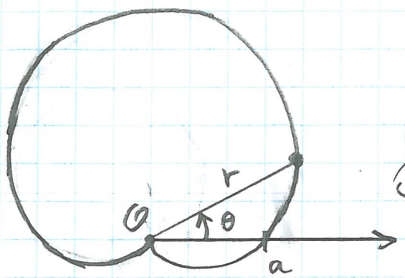


$r = a$



$r = 2a\cos\theta$

$(a > 0)$

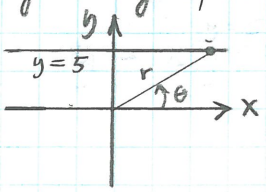


Kardioide

$r = a(1+\sin\theta)$

Omregning fra rektangulær ligning til polær ligning

Eks.



$$y = 5 \Leftrightarrow r\sin\theta = 5 \Leftrightarrow r = \frac{5}{\sin\theta}$$

$$\Leftrightarrow r = 5\csc\theta.$$