6. kursusgang : Repetition

Buelængde

Vektorfunktion

$$\vec{r}(t) = \begin{bmatrix} f(t) \\ g(t) \\ h(t) \end{bmatrix}, \quad t \in I$$

- $\vec{r}(t)$ differentiabel
- $\vec{r}\,'(t)$ kontinuert
- $\vec{r}\,'(t) \neq \vec{0}$ for alle $t \in I$

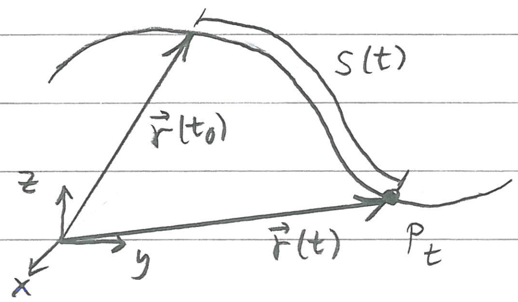
Tilhørende parametriserede kurve

$\overrightarrow{OP_t} = \vec{r}(t), \quad t \in I$

Buelængden regnet fra $t_0 \in I$ er

$s(t) = \int_{t_0}^{t} v(\tau)\, d\tau$,

hvor $v(t) = |\vec{r}\,'(t)|$ er farten.



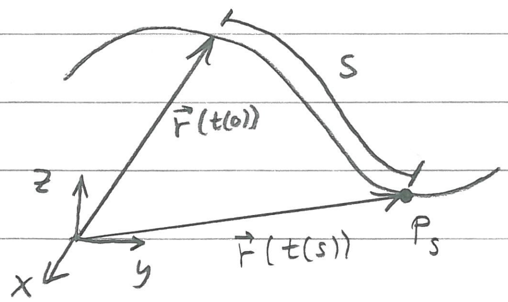
Bemærk at

$s'(t) = v(t)$

Da $s'(t) = |\vec{r}\,'(t)| > 0$ er $s(t)$ strengt voksende. Lad $t(s)$ betegne den inverse funktion.

Parametrisering ved buelængde s :

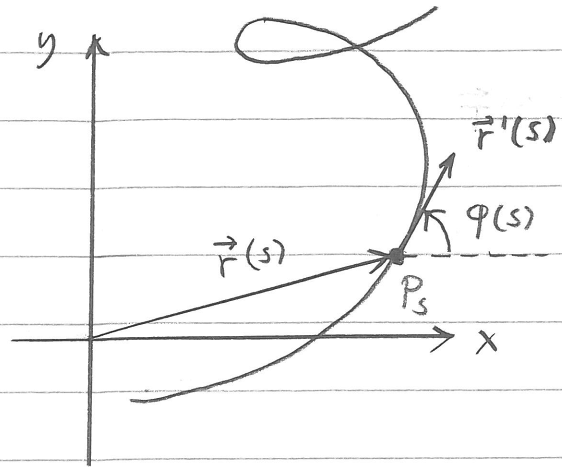
$\overrightarrow{OP_s} = \vec{r}\,(t(s))$



Plane kurvers krumning

Plan kurve parametriseret ved buelængde s :

$\overrightarrow{OP_s} = \vec{r}(s) = \begin{bmatrix} x(s) \\ y(s) \end{bmatrix}$.



Krumningen i punktet $P_s = (x(s), y(s))$ er
$\kappa(s) = |\varphi'(s)|$.

Formel for krumningen

Vektorfunktion
$\vec{r}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \quad t \in I$

- $\vec{r}(t)$ to gange diff.
- $\vec{r}\,'(t) \neq \vec{0}$ for alle $t \in I$

Tilhørende parametriserede kurve
$\overrightarrow{OP_t} = \vec{r}(t), \quad t \in I.$

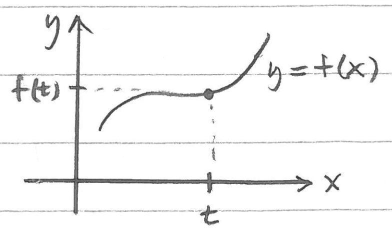
Krumningen i $P_t = (x(t), y(t))$ er

$$\boxed{\kappa(t) = \frac{|x'(t)y''(t) - x''(t)y'(t)|}{\left((x'(t))^2 + (y'(t))^2\right)^{3/2}}}$$

Specialtilfælde: $y = f(x)$
Parameterfremstilling: $x(t) = t, \; y(t) = f(t)$
Afledede $x'(t) = 1, \; y'(t) = f'(t), \; x''(t) = 0, \; y''(t) = f''(t)$

Krumningen i $P_t = (t, f(t))$ er

$$\kappa(t) = \frac{|f''(t)|}{\left(1 + (f'(t))^2\right)^{3/2}}.$$



Krumningen af en cirkel med radius $a > 0$ er $\kappa = \frac{1}{a}$.