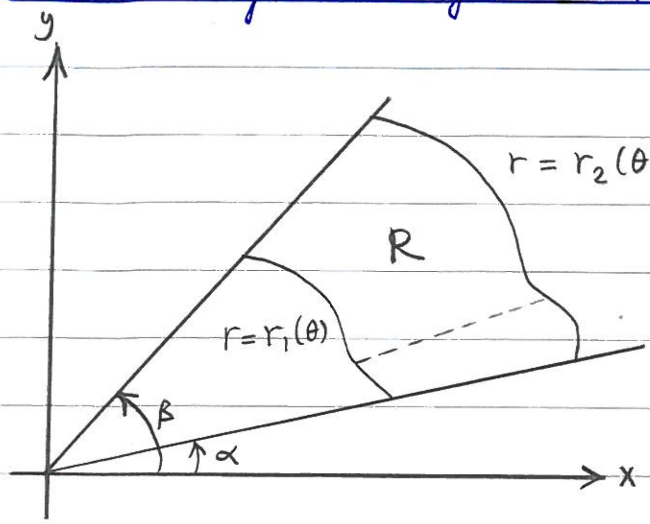
16. kursusgang : Repetition

Planintegralet i polære koordinater



$\alpha \le \beta \quad , \quad \beta - \alpha \le 2\pi .$

$0 \le r_1(\theta) \le r_2(\theta)$

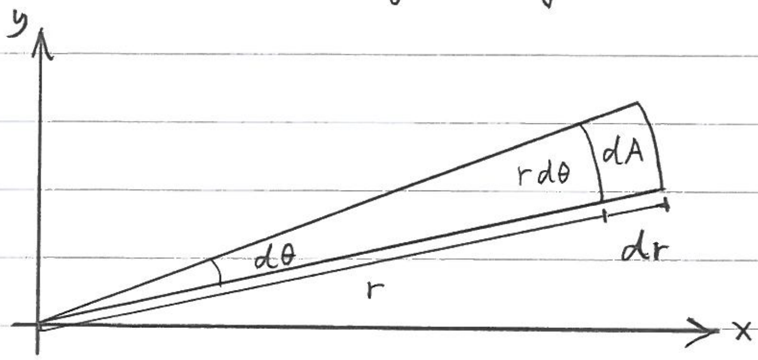
for alle $\theta \in [\alpha, \beta]$.

$$R = \{ (r,\theta) \mid \alpha \le \theta \le \beta \ , \ r_1(\theta) \le r \le r_2(\theta) \}$$

Et sådant område kaldes radialt simpel.

$$\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_{r_1(\theta)}^{r_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

Det ekstra r på højre side:



$dA \approx r\,d\theta\,dr$

$= r\,dr\,d\theta.$

$x = r\cos\theta$

$y = r\sin\theta$

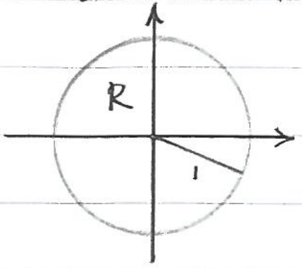
Sættes $f(x,y)=1$ fås en formel for arealet $a(R)$ af R:

$$a(R) = \int_\alpha^\beta \int_{r_1(\theta)}^{r_2(\theta)} r\,dr\,d\theta.$$

Opgaverne

§ 13.4   1. – 7. Beregn de indikerede arealer ved dobbelt integration i polære koordinater.

1. Arealet der er begrænset af cirklen med radius 1.

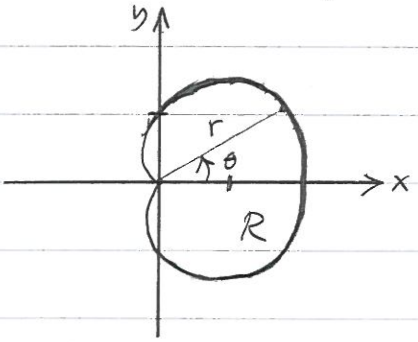


$R = \{(r,\theta) \mid 0 \leq \theta \leq 2\pi,\ 0 \leq r \leq 1\}$

Radialt simpel

$$a(R) = \int_0^{2\pi} \int_0^1 r\,dr\,d\theta = \ldots$$

3. Arealet begrænset af kardioiden $r = 1 + \cos\theta$.



$R = \{(r,\theta) \mid 0 \leq \theta \leq 2\pi,\ 0 \leq r \leq 1+\cos\theta\}$

Radialt simpel

$$a(R) = \int_0^{2\pi} \int_0^{1+\cos\theta} r\,dr\,d\theta = \ldots$$

Man får brug for en stamfunktion til $\cos^2(\theta)$.

Brug at $\cos(2\theta) = 2\cos^2(\theta) - 1$, så

$$\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}.$$

4. –

9. –

11. Brug integraltabellen i E&P.

$$\int_0^{\pi} \sin^4(\theta)\,d\theta = 2\int_0^{\pi/2} \sin^4(\theta)\,d\theta = 2 \cdot \frac{1\cdot 3}{2\cdot 4} \cdot \frac{\pi}{2} = \frac{3\pi}{8}$$