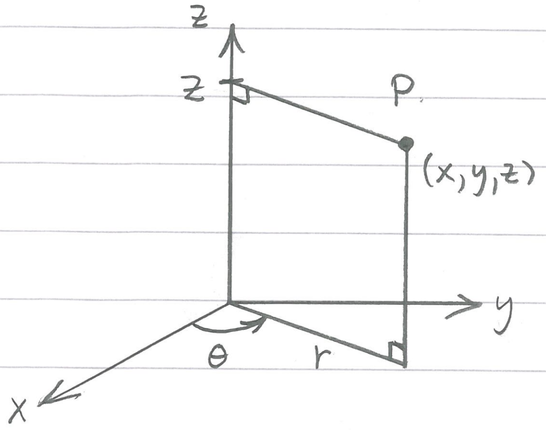
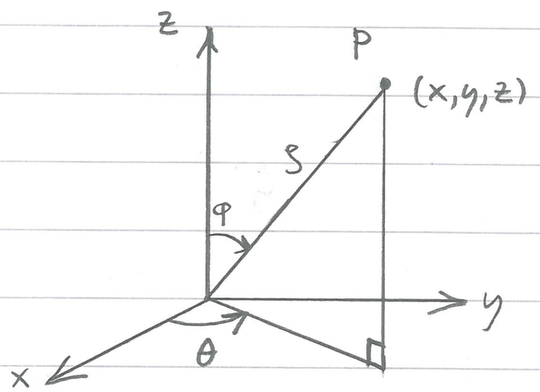
18. kursusgang: Repetition

Rumintegralet i cylinder- og sfæriske koordinater



P har cylinder koordinater (r, θ, z).

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$



P har sfæriske koordinater (ρ, φ, θ).

$$x = \rho\sin(\varphi)\cos(\theta)$$
$$y = \rho\sin(\varphi)\sin(\theta)$$
$$z = \rho\cos(\varphi)$$

$$\rho^2 = x^2 + y^2 + z^2$$

Simpelt område mht. cylinder koordinater:

$$T = \{(r,\theta,z) \mid \theta_1 \le \theta \le \theta_2,\ r_1(\theta) \le r \le r_2(\theta),\ z_1(r,\theta) \le z \le z_2(r,\theta)\}.$$

$$\iiint_T f(x,y,z)\,dV = \int_{\theta_1}^{\theta_2}\int_{r_1(\theta)}^{r_2(\theta)}\int_{z_1(r,\theta)}^{z_2(r,\theta)} f(r\cos\theta, r\sin\theta, z)\, r\,dz\,dr\,d\theta$$

Bemærk det ekstra r.

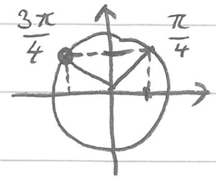
Sfærisk simpelt område

$$T = \{(\rho,\varphi,\theta) \mid \theta_1 \le \theta \le \theta_2,\ \varphi_1 \le \varphi \le \varphi_2,\ \rho_1(\varphi,\theta) \le \rho \le \rho_2(\varphi,\theta)\}.$$

$$\iiint_T f(x,y,z)\,dV =$$

$$\int_{\theta_1}^{\theta_2}\int_{\varphi_1}^{\varphi_2}\int_{\rho_1(\varphi,\theta)}^{\rho_2(\varphi,\theta)} f(\rho\sin\varphi\cos\theta, \rho\sin\varphi\sin\theta, \rho\cos\varphi)\,\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$

Eks. Find de rektangulære koordinater til punktet med cylinder koordinaterne $(r, \theta, z) = (2, \frac{3\pi}{4}, 3)$.



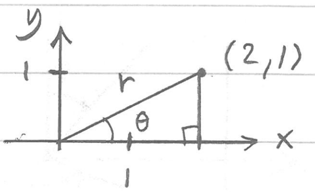
$$x = r\cos\theta = 2\cos\left(\frac{3\pi}{4}\right) = 2 \cdot \left(-\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$$

$$y = r\sin\theta = 2\sin\left(\frac{3\pi}{4}\right) = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$$

$$z = 3$$

Dvs. $(x, y, z) = \underline{\underline{(-\sqrt{2}, \sqrt{2}, 3)}}$

Eks. Find cylinder koordinater til punktet med rektangulære koordinater $(x, y, z) = (2, 1, -2)$.



$$r = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\tan\theta = \frac{1}{2}$$

$$\theta = \arctan\left(\frac{1}{2}\right)$$

Dvs. $(r, \theta, z) = \underline{\underline{(\sqrt{5}, \arctan(\tfrac{1}{2}), -2)}}$