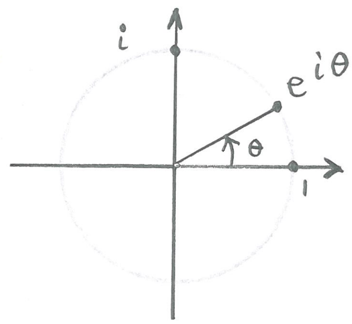
21. kursusgang : Repetition

Den komplekse eksponentialfunktion



Eulers ligning :

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

Den komplekse eksponentialfunktion

$$e^{\alpha + i\beta} = e^{\alpha} e^{i\beta} = e^{\alpha}(\cos\beta + i\sin\beta),$$

hvor $\alpha, \beta \in \mathbb{R}$.

Sætning

(1) $e^{z_1 + z_2} = e^{z_1} \cdot e^{z_2}$

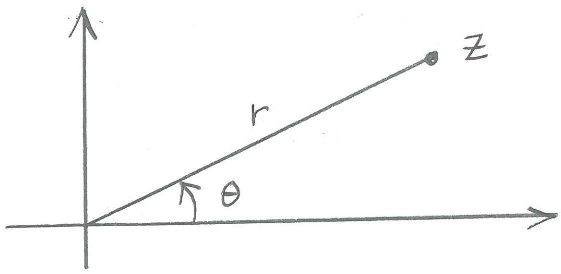
(2) $e^{z_1 - z_2} = \dfrac{e^{z_1}}{e^{z_2}}$ for alle $z_1, z_2 \in \mathbb{C}$.

(3) $\dfrac{d}{dt} e^{it} = i e^{it}$, $t \in \mathbb{R}$.

Polar form : $z = re^{i\theta}$, hvor

$r = |z|$;

$\theta = \arg(z)$.



Bemærk :

$\bar{z} = re^{-i\theta}$

Eks. $z = 1+i$.

$|z| = \sqrt{1^2+1^2} = \sqrt{2}$, $\mathrm{Arg}(z) = \frac{\pi}{4}$ så $z = \sqrt{2}\, e^{i\frac{\pi}{4}}$

De Moivre's formel :

$$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta), \quad n = 1, 2, 3, \ldots$$

Formler for cosinus og sinus :

$$\cos\theta = \mathrm{Re}(e^{i\theta}) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

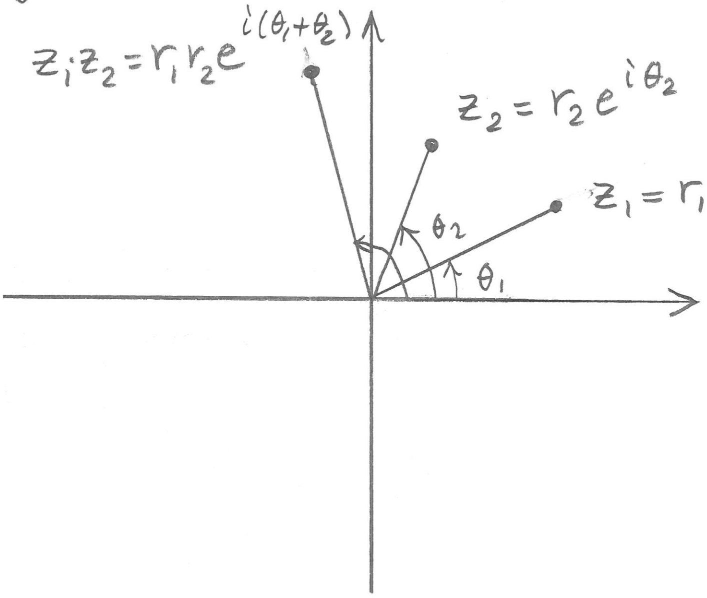
$$\sin\theta = \mathrm{Im}(e^{i\theta}) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

Pascal's trekant og binomialformlen

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1 \\
 & & & & \dots & & & &
\end{array}
$$

$$(a+b)^1 = a + b$$
$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$\dots$$

Geometrisk beskrivelse af kompleks multiplikation:



- $|z_1 \cdot z_2| = |z_1|\,|z_2|$
- $\arg(z_1 \cdot z_2) = \arg(z_1) + \arg(z_2)$