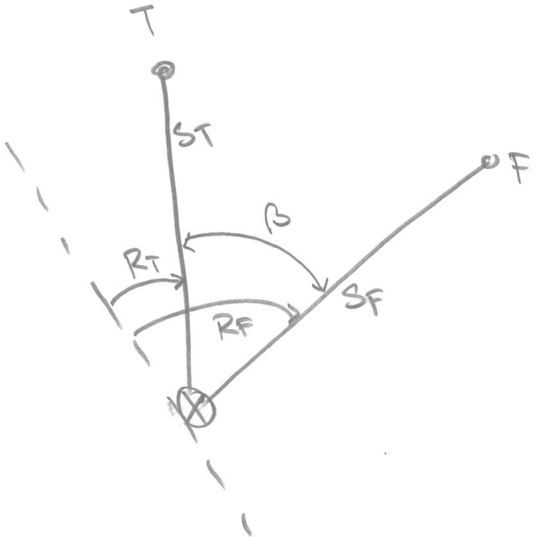
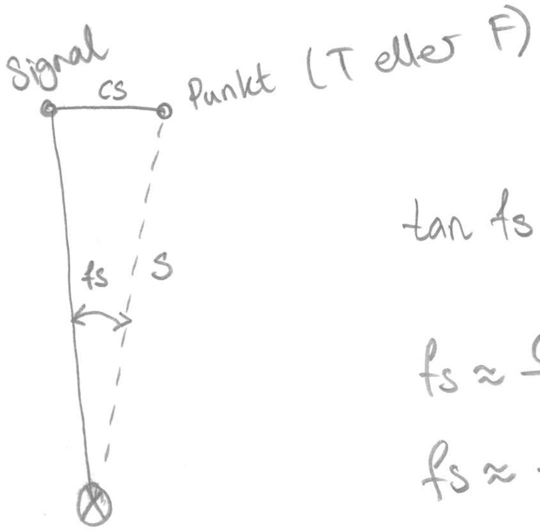


$$\beta = R_F - R_T$$

$$\sigma_\beta^2 = \left(\frac{\partial \beta}{\partial R_F}\right)^2 \sigma_{R_F}^2 + \left(\frac{\partial \beta}{\partial R_T}\right)^2 \sigma_{R_T}^2$$

Bemærk at $\left(\frac{\partial \beta}{\partial R_F}\right)^2 = \left(\frac{\partial \beta}{\partial R_T}\right)^2 = (\pm 1)^2 = 1.$



$$\tan fs = \frac{cs}{s}$$

for små vinkler gælder approximationen $\tan x \approx x$.

$$fs \approx \frac{cs}{s} \quad \text{i radian}$$

$$fs \approx \frac{cs}{s} w \quad \text{i gon idet } w = \frac{200}{\pi}.$$

$$\sigma_{R_F}^2 = \overset{\text{instrument}}{\left(\frac{\sigma_\sigma^2}{n_{hz}}\right)} + \overset{\text{centrerings fejl på instrument}}{\left(\frac{\sigma_{ci} w}{S_F}\right)^2} + \overset{\text{centrerings fejl på signal}}{\left(\frac{\sigma_{cs} w}{S_F}\right)^2}$$

$$\sigma_{fs}^2 = \left(\frac{\partial fs}{\partial cs}\right)^2 \sigma_{cs}^2 + \left(\frac{\partial fs}{\partial s}\right)^2 \sigma_s^2 = \left(\frac{w}{s}\right)^2 \sigma_{cs}^2 \left(\frac{cs\, w}{s^2}\right)^2 \sigma_s^2$$

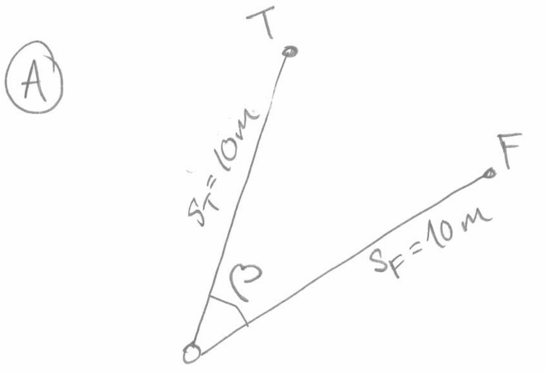
$$\approx \left(\frac{w \sigma_{cs}}{s}\right)^2$$ idet sidste led er ubetydeligt i praksis.

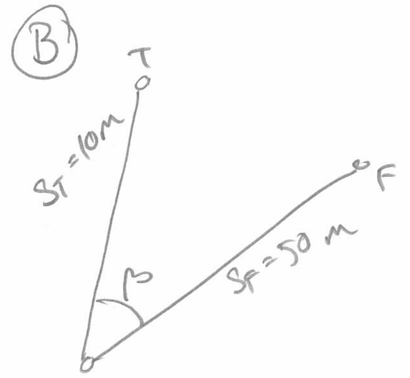
På tilsvarende måde kan det vises at

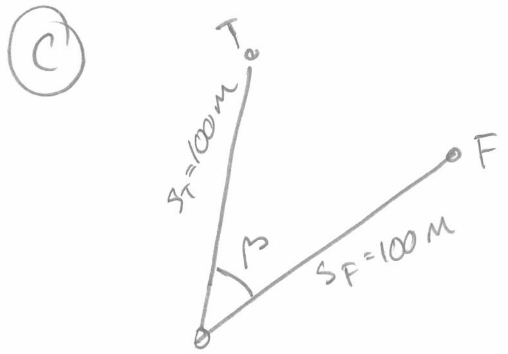
$$\sigma_{fi}^2 \approx \left(\frac{w \sigma_{ci}}{s}\right)^2.$$

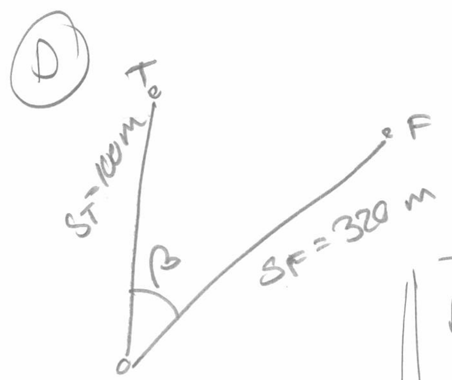
Eks 2.1

$\sigma_r = 0.001$ gon ; $n_{hz} = 1$ sats ; $\sigma_c = 0.005$ gon









Note til (2.5)

$$d_{\beta MAX} = \pm 3\sqrt{4}\,\sigma_r = \pm 6\,\sigma_r$$

± 3 kommer fra 99.9% fraktil i $N(0,1)$.

Den præcise grænse er 3.29, så ± 3 giver 99.7%

A: $\sigma_c = 0.033$ gon for både T og F.

jf. (2.4)

$$\sigma_\beta^2 = 2\left(\frac{\sigma_r^2}{n_{hz}}\right) + \left(\frac{\sigma_c \omega}{S_F}\right)^2 + \left(\frac{\sigma_c \omega}{S_T}\right)^2$$

$$= 2\left(\frac{0.001^2}{1}\right) + \left(\frac{0.005\,\frac{200}{\pi}}{10}\right)^2 + \left(\frac{0.005\,\frac{200}{\pi}}{10}\right)^2$$

$$\sigma_\beta = \sqrt{\text{-''-}} = 0.045 \text{ gon.}$$

B:

$$\sigma_\beta^2 = 2\left(\frac{0.001^2}{1}\right) + \left(\frac{0.005\,\frac{200}{\pi}}{50}\right)^2 + \left(\frac{0.005\,\frac{200}{\pi}}{10}\right)^2$$

$$\sigma_\beta = 0.032 \text{ gon}$$

C:

$$\sigma_\beta = \sqrt{2\left(\frac{0.001^2}{1}\right) + 2\left(\frac{0.005\,\frac{200}{\pi}}{100}\right)^2} = 0.0047$$

D:

$$\sigma_\beta = \sqrt{2(0.001^2) + \left(\frac{0.005\,\frac{200}{\pi}}{100}\right)^2 + \left(\frac{0.005\,\frac{200}{\pi}}{320}\right)^2} = 0.0036.$$

Bemærk:
Idet σ_V^2 angives i gon² skal de korrigeres med ω^{-2}, hvor $\omega = \frac{200}{\pi}$

$$S = (1+ppm_a 10^{-6}) S_d \sin V$$

$$\sigma_S^2 = \left(\frac{\partial S}{\partial ppm_a}\right)^2 \sigma_{ppm_a}^2 + \left(\frac{\partial S}{\partial S_d}\right)^2 \sigma_{S_d}^2 + \left(\frac{\partial S}{\partial V}\right)^2 \sigma_V^2$$

$$\frac{\partial S}{\partial ppm_a} = 10^{-6} S_d \sin V \quad ; \quad \frac{\partial S}{\partial S_d} = (1+ppm_a 10^{-6}) \sin V$$

$$\frac{\partial S}{\partial V} = (1+ppm_a 10^{-6}) S_d \cos V$$

$$\sigma_S^2 = \left[10^{-6} S_d \sin V\right]^2 \sigma_{ppm_a}^2 + \left[(1+ppm_a 10^{-6}) \sin V\right]^2 \sigma_{S_d}^2 + \left[(1+ppm_a 10^{-6}) S_d \cos V\right]^2 \frac{\sigma_V^2}{\omega^2}$$

Jvf. $\bar{X} = \frac{1}{n}\sum x_i \sim N\left(\mu, \frac{\sigma^2}{n}\right)$; så er $\sigma_V^2 = \sigma_V^2/n$ hvis n satser måles.

$$ppm_a = 278.96 - \left(\frac{0.3872 \times p \times 0.75}{1+0.003661\,t}\right)$$

$$\sigma_{ppm_a}^2 = \left(\frac{\partial ppm_a}{\partial p}\right)^2 \sigma_p^2 + \left(\frac{\partial ppm_a}{\partial t}\right)^2 \sigma_t^2 = \left(\frac{0.3872 \times 0.75}{1+0.003661\,t}\right)^2 \sigma_p^2 + \left(-\frac{0.003661 \times 0.3872 \times p \times 0.75}{(1+0.003661\,t)^2}\right)^2 \sigma_t^2$$

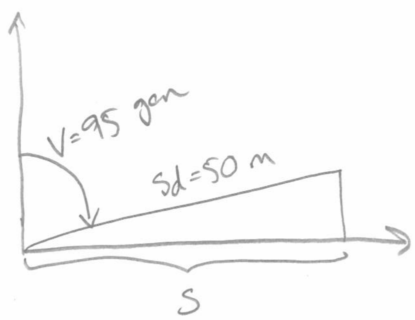
EKS 3.1

$\sigma_{ppm_a} = 2.9 \frac{mm}{km}$ for $t = 15$ °C; $\sigma_t = 1$; $p = 1013$ mbar og $\sigma_p = 10$ mbar

$\sigma_g = 0.005$ m; $\sigma_a = 0.005$; $\sigma_{ci} = 0.001$ m; $\sigma_{cp} = 0.005$ m

$\sigma_v = 0.001$ gon; $n_v = 1$ sats.

A



$$\sigma_S^2 = \underbrace{\left(10^{-6} S_d \sin V\right)^2 \sigma_{ppm_a}^2}_{} + \sin^2 V\left(\sigma_g^2 + (\sigma_a S_d 10^{-3})^2 + \sigma_{ci}^2 + \sigma_{cp}^2\right) + (S_d \cos V)^2 \frac{\sigma_v^2}{n_v w^2}$$

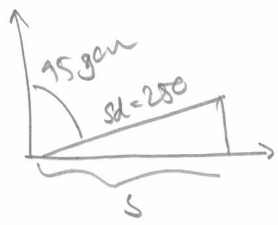
med overstående spredninger på σ_p og σ_t kan udtrykket negligeres.

$$+ \left[10^{-6} \times 50 \times \sin\left(\frac{95}{200}\pi\right)\right]^2 \left\{\left(\frac{\dots 1013}{15}\right)^2 1^2 + \left(\dots 15\right)^2 10^2\right\} = 2.115556 \times 10^{-8}$$

$$\sigma_S^2 = \sin^2\left(\frac{95}{200}\pi\right)\left[0.005^2 + (0.005 \times 50 \times 10^{-3})^2 + 0.001^2 + 0.005^2\right] + \left(50 \cos \frac{95}{200}\pi\right)^2 \frac{0.001^2}{\left(\frac{200}{\pi}\right)^2}$$

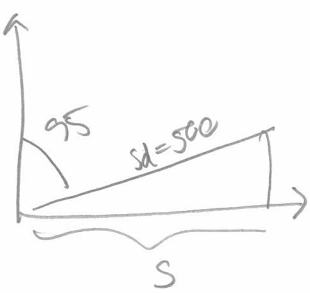
$\sigma_S = \sqrt{\;-''-\;} = \underline{0.00712}$

B



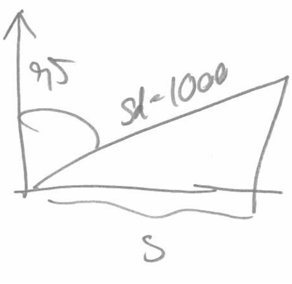
$\sigma_S = $ — 〃 — 250 — 250 $= \underline{0.00723}$

C



$\sigma_S = $ — 〃 — 500 — 〃 — 500 — 〃 — $= \underline{0.007707}$

D



$\sigma_S = $ — 1000 — 1000 — $= \underline{0.00925}$

Eks 3.2

$n_v = 1$; $\sigma_g = 0.005$ m ; $\sigma_a = 0.005 \frac{m}{km}$; $\sigma_{ci} = 0.001$ m ; $\sigma_{cp} = 0.005$ m ; $\sigma_v = 0.001$ gon

$$\sigma_s^2 = (\sin V)^2 \left[\sigma_g^2 + (\sigma_a S_d 10^{-3})^2 + \sigma_{ci}^2 + \sigma_{cp}^2\right] + (S_d \cos V)^2 \frac{\sigma_v^2}{n_v \omega^2}$$

A: $S_d = 250$ m; $V = 100$ gon.

$\sigma_s = 0.00725$ m

B: $S_d = 250$ m; $V = 95$ gon

$\sigma_s = 0.00723$ m

C: $S_d = 250$ m; $V = 90$ gon

$\sigma_s = 0.007187$ m

D: $S_d = 280$ m; $V = 50$ gon

$\sigma_s = 0.00583$ m