Øvelse 2.4 Kast med terning indtil 6'er

X : # kast indtil 6'er $\quad V = \{1, 2, 3, \ldots\}$

Sandsynlighedsfunktion for X:

$$f(1) = \frac{1}{6}$$

$$f(2) = P(\text{"ingen 6'er i 1. slag"} \cap \text{"6'er i 2. slag"}) = \frac{5}{6} \cdot \frac{1}{6}$$

(uafh.)

$$\vdots$$

$$f(x) = P(\text{"ingen 6'er i x-1 slag"} \cap \text{"6'er i x'te slag"}) = \underline{\left(\frac{5}{6}\right)^{x-1} \frac{1}{6}}$$

Øvelse 2.6 $0 < p < 1 \quad P(X=1) = 1 - P(X=0) = p \iff P(X=0) = 1 - p$

$$E(X) = 0 \cdot (1-p) + 1\,p = \underline{p}$$

$$V(X) = (0-p)^2(1-p) + (1-p)^2 p = (1-p)(p^2 + (1-p)p) = \underline{(1-p)\,p}$$

NB!! Svarer til $b(1, p)$

Øvelse 2.7 Kast med symmetrisk terning

X : # øjne $\quad V = \{1, 2, 3, 4, 5, 6\} \quad p(x) = \frac{1}{6} \quad \forall x \in V$

$$E(X) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \cdots + 6 \cdot \frac{1}{6} = \frac{21}{6} = \underline{3\tfrac{1}{2}}$$

$$V(X) = (1 - 3\tfrac{1}{2})^2 \tfrac{1}{6} + \cdots + (6 - 3\tfrac{1}{2})^2 \tfrac{1}{6} = \frac{2}{6}\left(\left(\frac{5}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right) = \frac{35}{12}$$

$$\sigma = \sqrt{\frac{35}{12}} \approx \underline{1{,}7078}$$

Øvelse 2.10 $P(\text{"arvelig lidelse"}) = 0{,}2 \quad n = 4$

X : # dyr med lidelse $\quad V = \{0, 1, 2, 3, 4\}$

$X \sim b(4, 0.2)$

$$P(X=0) = \binom{4}{0} 0.2^0\, 0.8^4 = 1 \cdot 0.8^4 = \underline{0.4096}$$

$$P(X=1) = \binom{4}{1} 0.2^1\, 0.8^3 = 4 \cdot 0.2 \cdot 0.8^3 = \underline{0.4096}$$

$$P(X=2) = \binom{4}{2} 0.2^2\, 0.8^2 = 6 \cdot 0.2^2 \cdot 0.8^2 = \underline{0.1536}$$

$$P(X=3) = \binom{4}{3} 0.2^3\, 0.8^1 = 4 \cdot 0.2^3 \cdot 0.8 = \underline{0.0256}$$

$$P(X=4) = \binom{4}{4} 0.2^4\, 0.8^0 = 1 \cdot 0.2^4 = \underline{0.0016}$$

KONTROL $\Sigma = 1$

Øvelse 2.12

$P(\text{"fejl"}) = 0.2 \qquad n = 10.000$

X : # kasserede enheder $\qquad X = \{0, 1, \dots, 10000\}$

$X \sim bn(10000, 0.2)$

$\mathbb{E}(X) = n \cdot p = \underline{\underline{2000}} \qquad \mathbb{V}(X) = n \cdot p(1-p) = 16000 \Rightarrow \underline{\underline{\sigma = 40}}$

Øvelse 2.15

$P(\text{" frø spirer "}) = 0.6$

$P(\text{" plante friske "} \mid \text{" frø spirer "}) = 0.5$

Bayes Formel: $P(A \cap B) = P(A|B)P(B)$

$\Downarrow$

$P(\text{" frø spirer " og " plante frisk "}) = 0.5 \cdot 0.6 = 0.3$

Sår 5 frø

X : # udkene friske planter

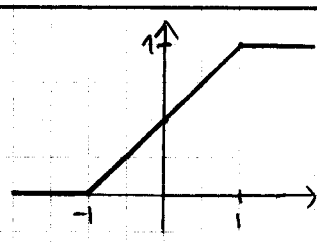
$X \sim bn(5, 0.3) \qquad V = \{0, 1, 2, 3, 4, 5\}$

$$P(X \geq 2) = 1 - P(X < 2) = 1 - (P(X=0) + P(X=1))$$
$$= 1 - \binom{5}{0} 0.3^0 0.7^5 - \binom{5}{1} 0.3^1 0.7^4$$
$$= 1 - 0.16807 - 0.36015$$
$$= \underline{\underline{0.47178}}$$

Øvelse 3.6

X ligefordelt på $[-1; 1]$

$$P(X \leq x) = \begin{cases} 0 & x \leq -1 \\ \frac{x+1}{2} & -1 < x < 1 \\ 1 & x \geq 1 \end{cases}$$



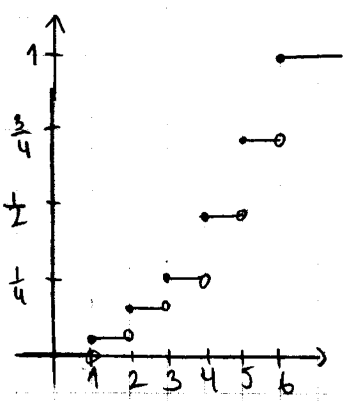
a) $P(X < 1/3) = \underline{\underline{\frac{2}{3}}}$ $\qquad$ c) $P(X=0) = \underline{\underline{0}}$

b) $P(X < -\frac{1}{2}) = \underline{\underline{\frac{1}{4}}}$ $\qquad$ d) $P(|X| < \frac{1}{3}) = P(-\frac{1}{3} < X < \frac{1}{3}) = P(X < \frac{1}{3}) - P(X < -\frac{1}{3}) = \frac{2}{3} - \frac{1}{3} = \underline{\underline{\frac{1}{3}}}$

Øvelse 3.7

Kast 2 gange med terning

X : største # øjne $\qquad V = \{1, 2, 3, 4, 5, 6\}$



$F(x) = P(X \leq x)$

$= P(\text{"1. kast \# øjne} \leq x\text{"} \cap \text{"2. kast \# øjne} \leq x\text{"})$

uafh. $\rightarrow$ $= P(\text{"1. kast \# øjne} \leq x\text{"}) \cdot P(\text{"2. kast \#} \leq x\text{"})$

$= \left(\frac{x}{6}\right)^2 \qquad x = 1, 2, 3, 4, 5, 6$