

Övning 3.18:

$$X \sim U(0, 1)$$

$$a) P(X \leq 0.6) = \underline{0.7257} \quad b) P(X \geq -1.5) = P(X \leq 1.5) = \underline{0.9332}$$

$$\begin{aligned} c) P(-1 \leq X \leq 2) &= P(X \leq 2) - P(X \leq -1) \\ &= P(X \leq 2) - (1 - P(X \leq 1)) \\ &= 0.9772 - 1 + 0.8413 = \underline{0.8185} \end{aligned}$$

Övning 3.20

$$X: \text{höjden} \quad X \sim U(176.3 \text{ cm}, 6.4^2 \text{ cm}^2)$$

1000 värnpliktiga beträgles - förväntade antal

$$\begin{aligned} a) \text{ över } 170 \text{ cm} \quad 1000 \cdot P(X > 170) &= 1000 \cdot (1 - P(X \leq 170)) \\ &= 1000 \cdot (1 - P(\underbrace{\frac{X-176.3}{6.4}}_{\sim U(0,1)} \leq \frac{170-176.3}{6.4})) \\ &= 1000 \cdot (1 - \Phi(\frac{-6.3}{6.4})) = 1000 \cdot \underbrace{\Phi(\frac{6.3}{6.4})}_{0.98} \\ &\approx \underline{837} \end{aligned}$$

$$\begin{aligned} b) \text{ över } 180 \text{ cm} \quad 1000 \cdot P(X > 180) &= 1000 \cdot (1 - P(X \leq 180)) \\ &= 1000 \cdot (1 - \Phi(\frac{180-176.3}{6.4})) \\ &= 1000 \cdot (1 - \underbrace{\Phi(\frac{3.7}{6.4})}_{0.58}) \\ &\approx \underline{281} \end{aligned}$$

$$\begin{aligned} c) \text{ över } 190 \text{ cm} \quad 1000 \cdot P(X > 190) &= 1000 \cdot (1 - \underbrace{\Phi(\frac{13.7}{6.4})}_{2.14}) \\ &\approx \underline{16} \end{aligned}$$

$$\begin{aligned} d) \text{ över } 200 \text{ cm} \quad 1000 \cdot P(X > 200) &= 1000 \cdot (1 - \underbrace{\Phi(\frac{23.7}{6.4})}_{3.7}) \\ &\approx \underline{0} \end{aligned}$$

Øvelse 3.21

X : tilvækst af kylling på en uge

$$X \sim \mathcal{N}(350 \text{ gr}, 30^2 \text{ gr}^2)$$

Betrægt 4 kyllinger, vægt tilvækst uafh.

$$a) P(X > 300) = 1 - P(X \leq 300) = 1 - \Phi\left(\frac{300 - 350}{30}\right) = 1 - \Phi\left(-\frac{5}{3}\right) = \Phi\left(\frac{5}{3}\right) = 0.9515$$

$$P(\text{"alle 4 tilvækst mere 300"}) = 0.9515^4 = \underline{0.8196}$$

$$b) P(X > 400) = 1 - \Phi\left(\frac{5}{3}\right) = 1 - 0.9515 = 0.0485 \Rightarrow P(X \leq 400) = 0.9515$$

$$P(\text{"min 1 tilvækst mere 400"}) = 1 - P(\text{"alle 4 tilvækst mindre 400"})$$

$$= 1 - 0.9515^4 = \underline{0.18}$$

Øvelse 3.19

$$X \sim \mathcal{N}(\mu, \sigma^2) \Rightarrow \frac{X - \mu}{\sigma} \sim \mathcal{N}(0, 1)$$

$$a) P(|X - \mu| \leq \sigma) = P(-\sigma \leq X - \mu \leq \sigma) = P\left(-1 \leq \frac{X - \mu}{\sigma} \leq 1\right)$$

$$= \Phi(1) - \underbrace{\Phi(-1)}_{1 - \Phi(1)} = 2\Phi(1) - 1 = \underline{0.682}$$

$$b) P(|X - \mu| \leq 2\sigma) \stackrel{\text{analog}}{=} 2\Phi(2) - 1 = \underline{0.9546}$$

$$c) P(|X - \mu| \leq 3\sigma) = 2\Phi(3) - 1 = \underline{0.9974}$$

