Eks. 2011 dw 14/12-12

apg. 1

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -2 & 0 \\ 0 & 2 & 1 & 0 & -3 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right]$$

Tjek $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

apg. 2

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

1) $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \xrightarrow{\text{ombyt}} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}$

$$\rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = R$$

2) $\det R = -1$ der var en rækkeombytn.
så $\det A = -(-1) = 1$

3) $\det((A^3)^T) = \det A^3 = (\det A)^3 = 1^3 = 1$

Opg. 3

1)
$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 1 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} \right\}$$

2)
$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

$$B = \left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$$

3).

rank $A = 2$ (spm1)

nullity $= 1$ (spm2)

Opg. 4

1) $\bar{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$$\bar{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} - \frac{2}{2} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\bar{v}_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix} - \frac{4}{2} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{1} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}$$

$$B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix} \right\}$$

2)

$$B' = \left\{ \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix} \right\}$$

Opg. 5

1)

$$\begin{vmatrix} 0-t & -2 \\ 2 & 4-t \end{vmatrix} = 0$$

$$\Updownarrow$$

$$t^2-4t+4=0$$

$$\Updownarrow$$

$$t = \frac{4 \pm \sqrt{16-16}}{2}$$

$$\Updownarrow$$

$$t = \underline{\underline{2}}$$

2)

$$\left[\begin{array}{cc|c} 0-2 & -2 & 0 \\ 2 & 4-2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} -2 & -2 & 0 \\ 2 & 2 & 0 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\underline{\underline{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}}}$$

3) A er <u>ej diagonaliserbar</u>, da dim. af egenrum er 1 men multiplicitet er to.

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Opg 6

$$W = \text{span}\left\{ \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}4\\5\end{bmatrix} \right\} \qquad (@)$$

1) $B = \left\{ \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}4\\5\end{bmatrix} \right\}$ er basis da

- vektorer i W,
- lineært uafh. da ej på linie,
- $|B| = 2 \geq \dim W$ ifølge (@)

2) $\bar{v} = \begin{bmatrix}3\\3\end{bmatrix} = -1 \begin{bmatrix}1\\2\end{bmatrix} + 1 \cdot \begin{bmatrix}4\\5\end{bmatrix}$ og derfor i W.

3) $[\bar{u}]_B = \begin{bmatrix}2\\-1\end{bmatrix}$

men så

$$\bar{u} = 2 \cdot \begin{bmatrix}1\\2\end{bmatrix} - 1 \cdot \begin{bmatrix}4\\5\end{bmatrix} = \underline{\underline{\begin{bmatrix}-2\\-1\end{bmatrix}}}$$

opg. 7

$$\begin{pmatrix} z_1 \\ z \end{pmatrix}' = \begin{pmatrix} 3z_1 \\ 2z_2 \end{pmatrix}$$

har løsn.

$$z_1 = a e^{3t}$$

$$z_2 = b e^{2t}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a e^{3t} \\ b e^{2t} \end{bmatrix}$$

$$= \begin{bmatrix} -2ae^{3t} - 3be^{2t} \\ ae^{3t} + 2be^{2t} \end{bmatrix}$$

$$\begin{cases} y_1(0) = -7 \\ y_2(0) = 4 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} -2a - 3b = -7 \\ a + 2b = 4 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ -2 & -3 & -7 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

6/9 $\quad a = 2 \quad b = 1$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} -4e^{3t} - 3e^{2t} \\ 2e^{3t} + 2e^{2t} \end{bmatrix}$$

Opg 8

1) $$T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 4 & 4 \end{bmatrix}$$

2) $$\begin{bmatrix} 1 & 3 & 4 \\ 2 & 4 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 19 \\ 22 \end{bmatrix}$$

3) Er surjektiv, da $\left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}\right\}$ lineært uafhængig

4) Er ej injektiv, da $\dim \mathbb{R}^3 = 3 > 2 = \dim \mathbb{R}^2$

opg. 9

Dimensionen af H er 3

opg. 10

V A er inverterbar

V lin. trans. er injektiv

V Er på reduceret echelon form

%.

V rank A = 5

%.

V −15 er en egenværdi

%.

%.

%.

V A er diagonaliserbar

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opg. 11

$n \geq 2$

$n = 3$

opg 12

V Enhver symmetrisk 4x4 matrix kan diagonaliseres

X.

X.

V

V