1 Fractions

Identifications

$$a = \frac{a}{1}; \qquad \frac{a}{b} = \frac{a \cdot c}{b \cdot c}$$

Addition

$$\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d}{b \cdot d} + \frac{c \cdot b}{d \cdot b} = \frac{a \cdot d + c \cdot b}{b \cdot d}$$

Subtraction

$$\frac{a}{b} - \frac{c}{d} = \frac{a \cdot d}{b \cdot d} - \frac{c \cdot b}{d \cdot b} = \frac{a \cdot d - c \cdot b}{b \cdot d}$$

Multiplication

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}; \qquad a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$$

Division

$$\frac{1}{(\frac{a}{b})} = \frac{b}{a}; \qquad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$$

$$\frac{(\frac{a}{b})}{c} = \frac{a}{b \cdot c}; \qquad \frac{a}{(\frac{b}{c})} = \frac{a \cdot c}{b}$$

Example

$$\frac{2}{3} + \frac{1}{7} = \frac{2 \cdot 7}{3 \cdot 7} + \frac{1 \cdot 3}{7 \cdot 3} = \frac{14}{21} + \frac{3}{21} = \frac{14 + 3}{21} = \frac{17}{21}$$

2 Exponents

If n is a positive integer, then

$$a^n = a \cdot a \cdot \ldots \cdot a \ (n \text{ factors}).$$

However, if a is positive, a^x is defined for any real number x.

$$a^0 = 1; \qquad a^{-1} = \frac{1}{a}$$

$$a^p \cdot a^q = a^{p+q}; \qquad \frac{a^p}{a^q} = a^{p-q}$$

$$(a \cdot b)^p = a^p \cdot b^p; \qquad (\frac{a}{b})^p = \frac{a^p}{b^p}$$

$$(a^p)^q = a^{p \cdot q}; \qquad a^{-p} = \frac{1}{a^p}$$

3 Radicals

Let n be a positive integer and a a real number.
If n is even, $\sqrt[n]{a}$ is defined for $a \geq 0$ by

$$\sqrt[n]{a} = b \quad \Leftrightarrow \quad b^n = a \text{ and } b \geq 0.$$

If n is odd, $\sqrt[n]{a}$ is defined for all a by

$$\sqrt[n]{a} = b \quad \Leftrightarrow \quad b^n = a.$$

Square root

$$\sqrt{a} = \sqrt[2]{a} = a^{\frac{1}{2}}$$

Examples

$$\sqrt{25} = 5; \qquad \sqrt[3]{-27} = -3.$$

Formulas

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}\,; \qquad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

4 Square Laws

$$(a+b)^2 = a^2 + b^2 + 2ab\,; \qquad (a+b)(a-b) = a^2 - b^2$$

5 The Quadratic Equation

$$ax^2 + bx + c = 0,$$

where $a \neq 0$. Discriminant

$$D = b^2 - 4ac$$

Roots

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

6 Trigonometric Functions

Radian measure: Angle measurement by arc length on the unit circle.

$$\pi \text{ rad } = 180^\circ$$

Trigonometric functions: The point with direction angle θ on the unit circle has coordinates $(\cos\theta, \sin\theta)$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}; \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}; \qquad \csc\theta = \frac{1}{\sin\theta}$$

The fundamental identity

$$\cos^2\theta + \sin^2\theta = 1$$

Symmetry identities

$$\begin{aligned} \cos(-\theta) &= \cos\theta; & \sin(-\theta) &= -\sin\theta \\ \cos(\pi+\theta) &= -\cos\theta; & \sin(\pi+\theta) &= -\sin\theta \\ \cos(\pi-\theta) &= -\cos\theta; & \sin(\pi-\theta) &= \sin\theta \\ \cos(\frac{\pi}{2}+\theta) &= -\sin\theta; & \sin(\frac{\pi}{2}+\theta) &= \cos\theta \\ \cos(\frac{\pi}{2}-\theta) &= \sin\theta; & \sin(\frac{\pi}{2}-\theta) &= \cos\theta \end{aligned}$$

Addition and subtraction formulas

$$\begin{aligned} \cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \\ \cos(\alpha-\beta) &= \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta) \\ \sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) \\ \sin(\alpha-\beta) &= \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta) \end{aligned}$$

Double-angle formulas

$$\begin{aligned} &\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1 \\ &\sin(2\theta) = 2\sin(\theta)\cos(\theta) \end{aligned}$$

Right triangles: For an acute angle θ in a right triangle, one has

$$\begin{aligned} \cos\theta &= \frac{adj}{hyp}; & \sin\theta &= \frac{opp}{hyp}; & \tan\theta &= \frac{opp}{adj} \\ \sec\theta &= \frac{hyp}{adj}; & \csc\theta &= \frac{hyp}{opp}; & \cot\theta &= \frac{adj}{opp} \end{aligned}$$

Exact values

	$30°$	$45°$	$60°$
	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

7 Exponential and Logarithmic Functions

Exponential function with base $a > 0$;

$$f(x) = a^x,\ x \in \mathbb{R}$$

When $a > 1$, the inverse function is the logarithmic function with base a;

$$\log_a(x),\ x \in \mathbb{R}_+$$

Laws of logarithms

$$\begin{aligned} \log_a(xy) &= \log_a(x) + \log_a(y); & \log_a(\frac{x}{y}) &= \log_a(x) - \log_a(y) \\ \log_a(x^y) &= y\log_a(x); & \log_a(\sqrt[n]{x}) &= \frac{1}{n}\log_a(x) \end{aligned}$$

Note that $\log_a(a) = 1$ and $\log_a(1) = 0$.
The logarithm function with base $e = 2.71828\ldots$ is the natural logarithm

$$\ln(x) = \log_e(x)$$

One has

$$a^x = e^{x\ln(a)}$$
$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

8 Derivatives

Differentiation rules

$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$
$$(c \cdot f(x))' = c \cdot f'(x)$$
$$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$
$$(\frac{f(x)}{g(x)})' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$
$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$
$$(f^{-1}(y))' = \frac{1}{f'(f^{-1}(y))}$$

Table of derivatives

$f(x)$	$f'(x)$
c	0
x	1
x^n	nx^{n-1}
$\sqrt{x} = x^{\frac{1}{2}}$	$\frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-\frac{1}{2}}$
$\frac{1}{x} = x^{-1}$	$-\frac{1}{x^2} = -x^{-2}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\sin(kx)$	$k\cos(kx)$
$\cos(kx)$	$-k\sin(kx)$
$\tan x$	$\sec^2 x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$
$\cot x$	$-\csc^2 x = -\frac{1}{\sin^2 x} = -1 - \cot^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
e^x	e^x
$\ln x$	$\frac{1}{x}$
e^{cx}	ce^{cx}
a^x	$a^x \ln a$

9 Integrals

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a),$$

where $F'(x) = f(x)$.

10 Summation

Notation

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$$

Rules of summation

$$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} 1 = n$$

Summation formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$$