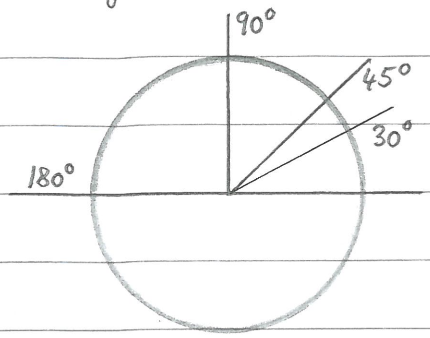
1. Session : Review of Trigonometry.

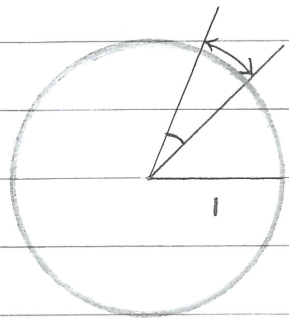
Angle measurement

- Degree measure :



- Radian measure :

Angle measurement by arc length on the unit circle



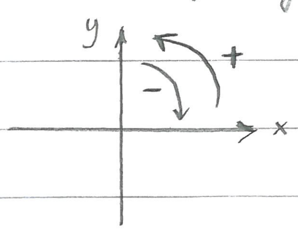
Recall: The circumference C of a circle of radius r is
$C = 2\pi r$.
For the unit circle, $r = 1$ and
$C = 2\pi \cdot 1 = 2\pi$.

rad.	2π	π	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$
deg.	360°	180°	90°	60°	45°	30°

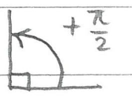
Since π rad $= 180°$, we have

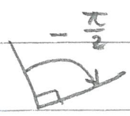
$$1 \text{ rad} = \frac{180°}{\pi} \qquad 1° = \frac{\pi}{180} \text{ rad}$$

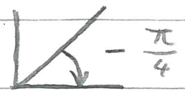
Positive / negative angles :



Ex.

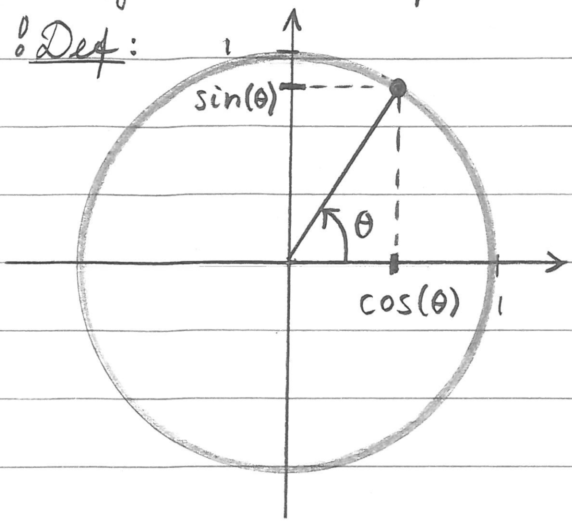






Trigonometric functions

! Def:



$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

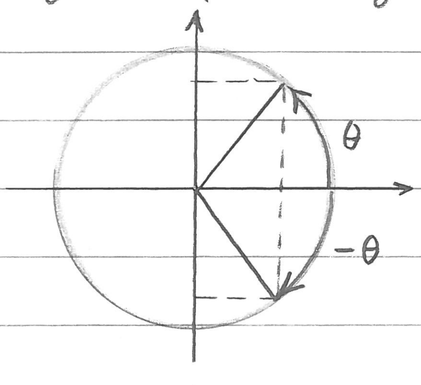
$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

Trigonometric identities

The square of the distance from $(0,0)$ to $(\cos(\theta), \sin(\theta))$ equals 1, so we have the fundamental identity:

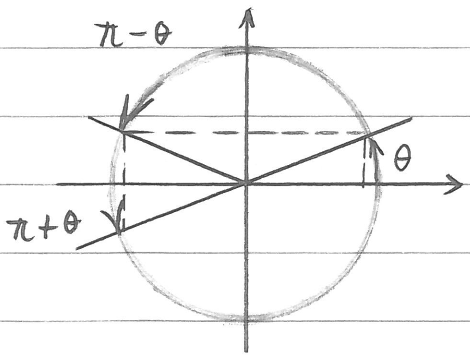
$$\cos^2(\theta) + \sin^2(\theta) = 1.$$

By the following figures, we find other identities:



$$\cos(-\theta) = \cos(\theta)$$

$$\sin(-\theta) = -\sin(\theta)$$



$$\cos(\pi+\theta) = -\cos(\theta)$$

$$\cos(\pi-\theta) = -\cos(\theta)$$

$$\sin(\pi+\theta) = -\sin(\theta)$$

$$\sin(\pi-\theta) = \sin(\theta)$$

Note also that

$$\cos(2\pi+\theta) = \cos(\theta)$$

$$\sin(2\pi+\theta) = \sin(\theta)$$

One has <u>addition and subtraction formulas</u> for sine and cosine

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

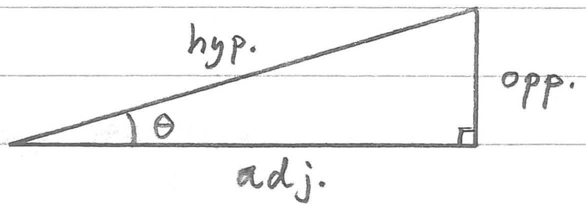
If we choose $+$ and take $\alpha = \beta = \theta$, we get the <u>double-angle formulas</u>

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 1 - 2\sin^2(\theta)$$

$$= 2\cos^2(\theta) - 1 ,$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta).$$

<u>Right triangles</u>



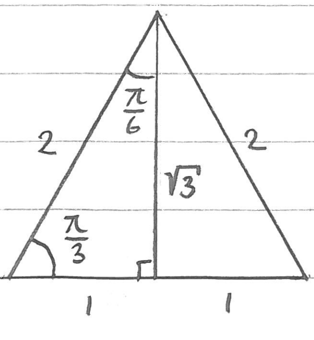
One can scale the triangle by $\frac{1}{hyp}$ and find :

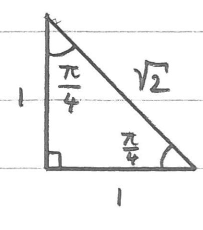
$$\cos(\theta) = \frac{adj}{hyp}, \qquad \sin(\theta) = \frac{opp}{hyp}, \qquad \tan(\theta) = \frac{opp}{adj},$$

$$\sec(\theta) = \frac{hyp}{adj}, \qquad \csc(\theta) = \frac{hyp}{opp}, \qquad \cot(\theta) = \frac{adj}{opp}.$$

<u>Exact values</u>

θ	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin(\theta)$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos(\theta)$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan(\theta)$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

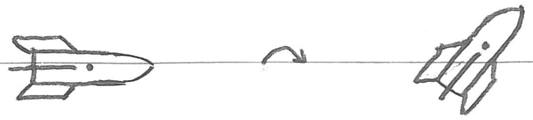




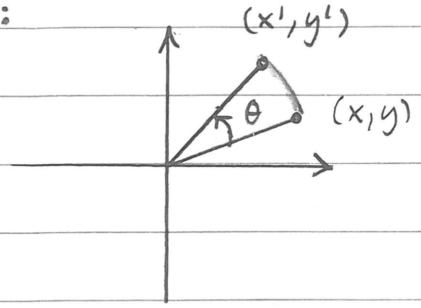
Applications

- Computer graphics

Rotation



2D:



$$x' = \cos(\theta)x - \sin(\theta)y$$
$$y' = \sin(\theta)x + \cos(\theta)y$$

3D: More complicated formulas involving sine and cosine

- Sound

Sinusoid $\quad x(t) = A\sin(\omega t + \varphi)$

t time / s

ω angular frequency / $\frac{\text{rad}}{\text{s}}$

φ phase / rad

A amplitude / V