Trial Exam 2011

Mathematics for Multimedia Applications
Medialogy

25. May 2011

Formalities

This trial exam set consists of 4 pages. There are 10 problems containing 32 sub-problems in total.

A number of points is indicated for every sub-problem. The sum of these points equals 100.

Date and time for the real exam: 1. June, 9:00 -13:00

You must indicate the following on each page:

- Full name
- Study number
- Page number

On the first page, you must indicate

- The total number of pages.

Problems

Problem 1.

1.a. (3 points) Differentiate the function $\sqrt{x}\sin(x)$.

1.b. (3 points) Let $f(x) = 3\sin(x^2 + 5)$. Calculate $f'(x)$.

Problem 2. Let $f(x) = 3\cos(2x)$.

2.a. (2 points) Calculate $f'(x)$.

2.b. (3 points) Find an x such that $f'(x) = 0$.

2.c. (5 points) Find all x such that $f'(x) = 0$.

Problem 3. Consider the function $y(t) = A\sin(\omega t)$, where A and ω are constants.

3.a. (4 points) Let $\omega = 1$. Sketch the graph of $y(t)$, $0 \leq t \leq 2\pi$ for $A = 1$ and $A = 2$ in the same (t, y)-coordinate system.

3.b. (4 points) Let $A = 1$. Sketch the graph of $y(t)$, $0 \leq t \leq 2\pi$ for $\omega = 1$ and $\omega = 2$ in a new (t, y)-coordinate system.

3.c. (4 points) Calculate $y'(t)$ and $y''(t)$ for the function $y(t) = A\sin(\omega t)$.

Problem 4. Evaluate the following integrals:

4.a. (3 points) $\int_{-1}^{1}(3x^2 + 8x + 1)\,\mathrm{d}x$.

4.b. (3 points) $\int_{0}^{\pi/2}(\cos(x) + 2)\,\mathrm{d}x$.

4.c. (3 points) $\int_{1}^{3}\frac{1}{x}\,\mathrm{d}x$.

Problem 5. Let P, Q and R be three points in 3D-space; P has coordinates $(1, 2, 0)$, Q has coordinates $(1, 4, 0)$ and R has coordinates $(4, 4, \sqrt{3})$.

5.a. (2 points) Find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

5.b. (2 points) Write parametric equations of the line that passes through P and Q.

5.c. (4 points) Find the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

Problem 6. Let P, Q and R be three points in 3D-space; P has coordinates $(7, 2, 3)$, Q has coordinates $(8, 2, 2)$ and R has coordinates $(9, 5, 4)$.

6.a. (2 points) Find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

6.b. (3 points) Compute the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$.

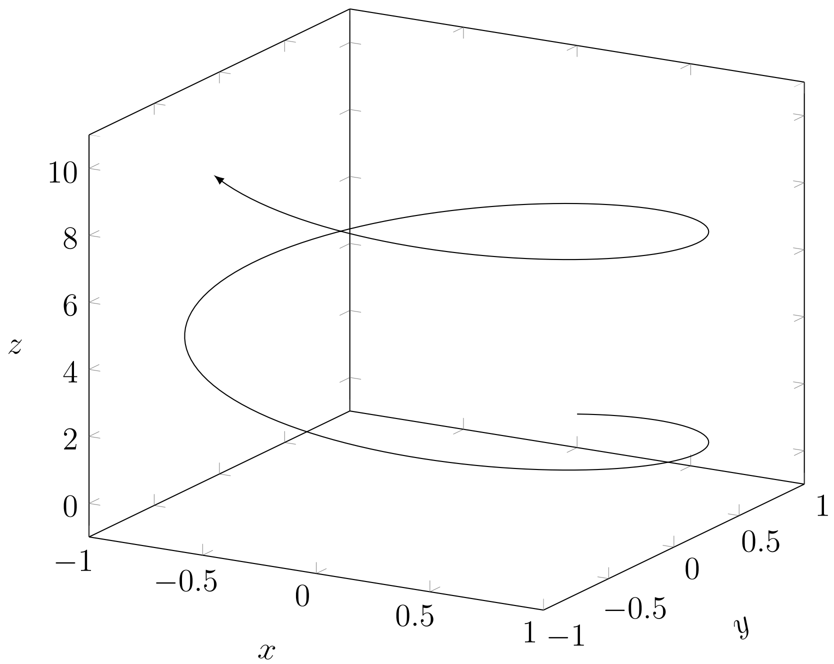
6.c. (3 points) Find the area of the triangle with vertices P, Q and R.

6.d. (3 points) Find an equation for the plane through P, Q and R.

Problem 7. Consider a moving point in 3D-space with position vector given by

$$\vec{r}(t) = (\sin(t), \cos(t), t)$$

Here is a plot of the associated curve for the time interval $0 \le t \le 10$:



7.a. (3 points) Compute the velocity vector $\vec{v}(t)$ and the speed $v(t) = |\vec{v}(t)|$.

7.b. (2 points) Compute the acceleration vector $\vec{a}(t)$.

7.c. (2 points) Compute the angle between $\vec{v}(t)$ and $\vec{a}(t)$.

Problem 8. Consider the following system of linear equations:

$$\begin{aligned} x_1 + x_2 - 6x_3 &= 3 \\ x_2 + x_3 &= 2 \\ 2x_1 + x_2 - 13x_3 &= 4 \end{aligned}$$

8.a. (2 points) Find the augmented matrix of the system.

8.b. (4 points) Find a row echelon form of the augmented matrix and mark the pivot positions.

8.c. (3 points) Find the reduced row echelon form of the augmented matrix.

8.d. (4 points) Write down the general solution of the system.

Problem 9. Define three matrices as follows:

$$A = \begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

9.a. (3 points) Compute the product AB.

9.b. (3 points) Compute the product BA.

9.c. (4 points) Find $A^T B^T$.

9.d. (5 points) Determine whether C is invertible. If so, find its inverse.

Problem 10. Consider the unit square with vertices $(0,0)$, $(1,0)$, $(0,1)$ and $(1,1)$ in an (x,y)-coordinate system. For each of the following three matrix transformations, sketch the image of this unit square.

10.a. (3 points) $T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

10.b. (3 points) $T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

10.c. (3 points) $T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$