Theorem: *For any real number $x \neq 1$ one has that*

$$\sum_{n=0}^{k} x^n = \frac{1 - x^{k+1}}{1 - x}$$

Proof. Let s denote the sum on the left hand side. We have

$$s = 1 + x + x^2 + x^3 + \cdots + x^k$$

such that

$$xs = x + x^2 + x^3 + x^4 + \cdots + x^{k+1}.$$

Thus,

$$(1 - x)s = s - xs = 1 - x^{k+1}$$

such that

$$s = \frac{1 - x^{k+1}}{1 - x}$$

□

Corollary: *If $|x| < 1$, then*

$$\sum_{n=0}^{\infty} x^n := \lim_{k \to \infty} \sum_{n=0}^{k} x^n = \frac{1}{1 - x}$$