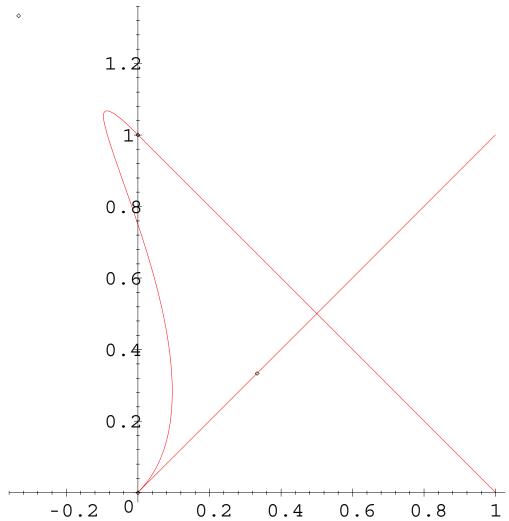
Løsningsskitse:

1 $Q_0 = [0,0]$, $Q_3 = [3,5]$; Q_1, Q_2 skal ligge på liniestykket mellem Q_0 og Q_3.

2 $\overrightarrow{p}(t) = [2t^3 - 3t^2 + t, -2t^3 + 2t^2 + t]$.
$Q_0 = [0,0]$, $Q_1 = [\frac{1}{3}, \frac{1}{3}]$, $Q_2 = [-\frac{1}{3}, \frac{4}{3}]$, $Q_3 = [0,1]$.



Figur 1: Kubisk kurve, hastighedsvektorer og Bézierpunkter

3 Ved differentiation af Bèzierpolynomierne $B_{i,3}$ finder man følgende tabel:

i	0	1	2	3
$B'_{i,3}(0)$	-3	3	0	0
$B'_{i,3}(1)$	0	0	-3	3
$B''_{i,3}(0)$	6	-12	6	0
$B''_{i,3}(1)$	0	6	-12	6

Med $B(t) = B_{0,3}(t)Q_0 + B_{1,3}(t)Q_1 + B_{2,3}(t)Q_2 + B_{3,3}(t)Q_3$ beregnes krumningen som $\kappa(t) = \frac{[B'(t), B''(t)]}{|B'(t)|^3}$. Ved indsætning fås:

- $$\kappa(0) = \frac{[3\overrightarrow{Q_0Q_1}, 6(\overrightarrow{Q_1Q_0} + \overrightarrow{Q_1Q_2})]}{27|\overrightarrow{Q_0Q_1}|^3} = \frac{2}{3}\frac{[\overrightarrow{Q_0Q_1}, \overrightarrow{Q_1Q_2}]}{|\overrightarrow{Q_0Q_1}|^3}.$$

- $$\kappa(1) = \frac{[3\overrightarrow{Q_2Q_3}, 6(\overrightarrow{Q_2Q_1} + \overrightarrow{Q_2Q_3})]}{27|\overrightarrow{Q_2Q_3}|^3} = \frac{2}{3}\frac{[\overrightarrow{Q_2Q_3}, \overrightarrow{Q_2Q_1}]}{|\overrightarrow{Q_2Q_3}|^3}.$$

4 1. $\mathbf{r}(t) = [\cos^3(t), \sin^3(t)]$, $\mathbf{r}'(t) = [-3\sin(t)\cos^2(t), 3\cos(t)\sin^2(t)] =$
$= 3\sin(t)\cos(t)[-\cos(t), \sin(t)]$. $v(t) = \| \mathbf{r}'(t) \| = 3\sin(t)\cos(t)$.

2. $l = \int_0^{\frac{\pi}{2}} v(t)dt = 3\int_0^{\frac{\pi}{2}} \sin(t)\cos(t)\; dt = \frac{3}{2}[\sin^2(t)]_0^{\frac{\pi}{2}} = \frac{3}{2} < \frac{\pi}{2} = L.$

3. $[\mathbf{r}'(t), \mathbf{r}''(t)] = 3\sin(t)\cos(t)[-\cos(t), \sin(t)] \times 3[\cos(t)(3\sin^2(t) - 1), \sin(t)(3\cos^2(t) - 1)] = -9\sin^2(t)\cos^2(t)(3\cos^2(t) - 1 + 3\sin^2(t) - 1) = -9\sin^2(t)\cos^2(t)$.

$$\kappa(t) = \frac{\| \mathbf{r}'(t) \times \mathbf{r}''(t) \|}{v(t)^3} = \frac{-9\sin^2(t)\cos^2(t)}{27\sin^3(t)\cos^3(t)} = -\frac{1}{3\sin(t)\cos(t)} = -\frac{2}{3\sin(2t)}.$$

$|\kappa(t)|$ er mindst når nævneren $\sin(2t)$ er mindst. Det sker for $t = \frac{\pi}{4}$, og $\kappa(\frac{\pi}{4}) = -\frac{2}{3}$.

4. $\mathbf{v}(\frac{\pi}{4}) = \frac{3}{4}\sqrt{2}[-1, 1]$, $\mathbf{a}(\frac{\pi}{4}) = \frac{3}{4}\sqrt{2}[1, 1]$, $\mathbf{v}(\frac{\pi}{4}) \cdot \mathbf{a}(\frac{\pi}{4}) = 0$.