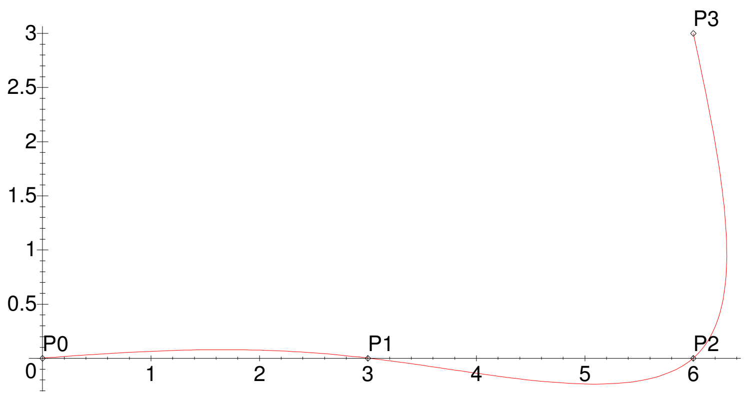
Løsningsskitse

2 1. Check, at $\mathbf{A}_4\mathbf{C} = 3\mathbf{I}_4$.

2. $\mathbf{v}_0 = [2.8, 0.2], \mathbf{v}_1 = [3.4, -0.4], \mathbf{v}_2 = [1.6, 1.4], \mathbf{v}_3 = [-0.8, 3.8]$.

3.

$$\begin{aligned}\mathbf{p}_1(t) &= [0.2t^3 + 2.8t, -0.2t^3 + 0.2t],\\ \mathbf{p}_2(t) &= [-t^3 + 0.6t^2 + 3.4t + 3, t^3 - 0.6t^2 - 0.4t],\\ \mathbf{p}_3(t) &= [0.8t^3 - 2.4t^2 + 1.6t + 6, -0.8t^3 + 2.4t^2 + 1.4t].\end{aligned}$$



Figur 1: Kubisk spline gennem 4 punkter

3 1. $\mathbf{r}(t) = [t, t^2, \frac{2}{3}t^3]$, $\mathbf{r}'(t) = [1, 2t, 2t^2]$, $\mathbf{r}''(t) = [0, 2, 4t]$, $\mathbf{r}'''(t) = [0, 0, 4]$.

(a) $v(t) = \| \mathbf{r}'(t) \| = \sqrt{1 + 4t^2 + 4t^4} = 1 + 2t^2$.

(b) $\mathbf{r}'(t) \times \mathbf{r}''(t) = [1, 2t, 2t^2] \times [0, 2, 4t] = [4t^2, -4t, 2]$,
$\| \mathbf{r}'(t) \times \mathbf{r}''(t) \| = \sqrt{16t^4 + 16t^2 + 4} = 4t^2 + 2 = 2(1 + 2t^2)$.

(c) $\kappa(t) = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{v(t)^3} = \frac{2(2t^2+1)}{(2t^2+1)^3} = \frac{2}{(2t^2+1)^2}$,
$\tau(t) = \frac{\mathbf{r}'''(t) \cdot (\mathbf{r}'(t) \times \mathbf{r}''(t))}{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|^2} = \frac{[0,0,4] \cdot [4t^2, -4t, 2]}{(4t^2+2)^2} = \frac{2}{(2t^2+1)^2}$.

(d) $\mathbf{t}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} = \frac{1}{1+2t^2}[1, 2t, 2t^2]$.
Lad $\phi(t)$ betegne vinklen mellem $\mathbf{t}(t)$ og $\mathbf{a}$. Så gælder:
$\cos\phi(t) = \frac{\mathbf{t}(t) \cdot \mathbf{a}}{\|\mathbf{t}(t)\| \|\mathbf{a}\|} = \frac{[1, 2t, 2t^2] \cdot [1, 0, 1]}{(1+2t^2)\sqrt{2}} = \frac{1+2t^2}{(1+2t^2)\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \phi(t) = \frac{\pi}{4}$.

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