Kurver og flader i geometri, arkitektur og design 4. lektion

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Krumning - plane kurver; definition

- $\mathbf{T}(s) = (\cos\varphi(s), \sin\varphi(s))$ – den vandrende enhedstangentvektor; retning givet ved vinklen $\varphi(s)$ i forhold til X-aksen.
- $\mathbf{N}(s) = \hat{\mathbf{T}}(s) = (-\sin\varphi(s), \cos\varphi(s))$ – den vandrende enhedsnormalvektor.
- $\mathbf{T}'(s)$ måler vinkelhastigheden $\varphi'(s)$ og er vinkelret på $\mathbf{T}(s)$:
  - $\mathbf{T}'(s) = \varphi'(s)(-\sin\varphi(s), \cos\varphi(s)) = \varphi'(s)\mathbf{N}(s)$
- Definition: Krumning $\kappa(s) = \varphi'(s)$ i $\mathbf{r}(s)$: $\mathbf{T}'(s) = \kappa(s)\mathbf{N}(s)$

Krumning - plane kurver; beregning

- $\mathbf{v}(t) = \mathbf{r}'(t) = v(t)\mathbf{T}(t)$
- $\mathbf{T}'(t) = \frac{d}{dt}\mathbf{T}(t) = \frac{d}{ds}\mathbf{T}(t) \cdot \frac{ds}{dt} = \kappa(t)v(t)\mathbf{N}(t)$
- $\mathbf{a}(t) = \mathbf{r}''(t) = v'(t)\mathbf{T}(t) + v(t)\mathbf{T}'(t) =$ $v'(t)\mathbf{T}(t) + \kappa(t)v^2(t)\mathbf{N}(t)$ – tangential og normal komponent af accelerationsvektoren
- Projektionen på normalvektoren ved prikprodukt med $\mathbf{N}(t)$: $\kappa(t)v^2(t) = \mathbf{a}(t) \cdot \mathbf{N}(t) = \mathbf{a}(t) \cdot \frac{1}{v(t)}\hat{\mathbf{r}}'(t)$
- $$\begin{aligned} \kappa(t) &= \frac{1}{v^3(t)}\mathbf{r}''(t) \cdot \hat{\mathbf{r}}'(t) \\ &= \frac{1}{(x'(t)^2 + y'(t)^2)^{\frac{3}{2}}}(x''(t), y''(t)) \cdot (-y'(t), x'(t)) \\ &= \frac{x'(t)y''(t) - x''(t)y'(t)}{(x'(t)^2 + y'(t)^2)^{\frac{3}{2}}} \end{aligned}$$

Rumkurver

I rummet er der en hel plan af normalretninger i hvert punkt.
Krumning $\kappa(s) = |\mathbf{T}'(s)|$.
Hovednormalvektor $\mathbf{N}(s) = \frac{\mathbf{T}'(s)}{\kappa(s)}$.

$$\begin{aligned}
\mathbf{a}(t) = \mathbf{r}''(t) &= v'(t)\mathbf{T}(t) + v(t)\mathbf{T}'(t) \\
&= v'(t)\mathbf{T}(t) + \kappa(t)v^2(t)\mathbf{N}(t) \\
\mathbf{v}(T) \times \mathbf{a}(t) &= v(t)\mathbf{T}(t) \times (v'(t)\mathbf{T}(t) + \kappa(t)v^2(t)\mathbf{N}(t)) \\
&= \mathbf{0} + v^3(t)\kappa(t)(\mathbf{T}(t) \times \mathbf{N}(t)) \\
|\mathbf{v}(T) \times \mathbf{a}(t)| &= v^3(t)\kappa(t) \\
\kappa(t) &= \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.
\end{aligned}$$

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