Prikproduktregel 1

Theorem

$f, g : V \to \mathbf{R}^m, V \subset \mathbf{R}^m$ differentiabel i $\mathbf{a} \Rightarrow f \cdot g : V \to \mathbf{R}$ differentiabel i $\mathbf{a}$ og $\nabla(f \cdot g)(a) = g(\mathbf{a})Df(\mathbf{a}) + f(\mathbf{a})Dg(\mathbf{a})$.

$$f(\mathbf{a}+\mathbf{h}) \cdot g(\mathbf{a}+\mathbf{h}) - f(\mathbf{a}) \cdot g(\mathbf{a}) - g(\mathbf{a})Df(\mathbf{a})(\mathbf{h}) - f(\mathbf{a})Dg(\mathbf{a})(\mathbf{h}) =$$
$$(f(\mathbf{a}+\mathbf{h}) - f(\mathbf{a}) - Df(\mathbf{a})(\mathbf{h})) \cdot g(\mathbf{a}+\mathbf{h})$$
$$+Df(\mathbf{a})(\mathbf{h}) \cdot (g(\mathbf{a}+\mathbf{h}) - g(\mathbf{a}))$$
$$+f(\mathbf{a}) \cdot (g(\mathbf{a}+\mathbf{h}) - g(\mathbf{a}) - Dg(\mathbf{a})(\mathbf{h}))$$

Prikproduktregel 2

Theorem

$f, g : V \to \mathbf{R}^m, V \subset \mathbf{R}^m$ *differentiabel i* $\mathbf{a} \Rightarrow f \cdot g : V \to \mathbf{R}$ *differentiabel i* $\mathbf{a}$ *og* $\nabla(f \cdot g)(a) = g(\mathbf{a})Df(\mathbf{a}) + f(\mathbf{a})Dg(\mathbf{a})$.

$$f(\mathbf{a}+\mathbf{h}) \cdot g(\mathbf{a}+\mathbf{h}) - f(\mathbf{a}) \cdot g(\mathbf{a}) - g(\mathbf{a})Df(\mathbf{a})(\mathbf{h}) - f(\mathbf{a})Dg(\mathbf{a})(\mathbf{h}) =$$

$$(f(\mathbf{a}+\mathbf{h}) - f(\mathbf{a}) - Df(\mathbf{a})(\mathbf{h})) \cdot g(\mathbf{a}+\mathbf{h}) \quad T_1(\mathbf{h}) := \varepsilon(\mathbf{h}) \cdot g(\mathbf{a}+\mathbf{h})$$

$$+Df(\mathbf{a})(\mathbf{h}) \cdot (g(\mathbf{a}+\mathbf{h}) - g(\mathbf{a})) \quad T_2(\mathbf{h})$$

$$+f(\mathbf{a}) \cdot (g(\mathbf{a}+\mathbf{h}) - g(\mathbf{a}) - Dg(\mathbf{a})(\mathbf{h})) \quad T_3(\mathbf{h}) := f(\mathbf{a}) \cdot \delta(\mathbf{h})$$

$$|T_1(\mathbf{h})| \leq \| \varepsilon(\mathbf{h}) \| \| g(\mathbf{a}+\mathbf{h}) \|$$

$$0 \leq \frac{\|T_1(\mathbf{h})\|}{\|\mathbf{h}\|} \leq \frac{\|\varepsilon(\mathbf{h})\|}{\|\mathbf{h}\|} \| g(\mathbf{a}+\mathbf{h}) \| \to_{\mathbf{h}\to\mathbf{0}} 0 \| g(\mathbf{a}) \| = 0.$$

$$|T_2(\mathbf{h})| \leq \| Df(\mathbf{a})(\mathbf{h}) \| \| g(\mathbf{a}+\mathbf{h}) - g(\mathbf{a}) \|$$

$$\leq \| Df(\mathbf{a}) \| \| \mathbf{h} \| \| g(\mathbf{a}+\mathbf{h}) - g(\mathbf{a}) \|$$

$$0 \leq \frac{\|T_2(\mathbf{h})\|}{\|\mathbf{h}\|} \leq \| Df(\mathbf{a}) \| \| g(\mathbf{a}+\mathbf{h}) - g(\mathbf{a}) \| \to_{\mathbf{h}\to\mathbf{0}} \| Df(\mathbf{a}) \| 0 = 0.$$