Matematisk Analyse I 14. lektion

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19.11.2009

Approksimation af grafen for en differentiabel funktion $f : O \to \mathbf{R}$, $O \subseteq \mathbf{R}^2$ gennem tangentplan i punktet $(x_0, y_0, f(x_0, y_0)) \in \mathbf{R}^3$ givet ved

parameterfremstilling $z=g(x,y) =$
$$f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0);$$

normalvektor $\eta = (-\frac{\partial f}{\partial x}(x_0, y_0), -\frac{\partial f}{\partial y}(x_0, y_0), 1)$.

Differentiable vektorfunktioner

Vektorfunktion $F = (F_1, \dots, F_m) : O \subseteq \mathbf{R}^n \to \mathbf{R}^m$.
Affin afbildning $G : \mathbf{R}^n \to \mathbf{R}^m$, $G(\mathbf{x}) = \mathbf{c} + A(\mathbf{x})$, A lineær.
Jacobi matrix i $\mathbf{x}_0 \in O$:

$$DF(\mathbf{x}_0) = \begin{bmatrix} \nabla F_1(\mathbf{x}_0) \\ \dots \\ \nabla F_m(\mathbf{x}_0) \end{bmatrix} = \begin{bmatrix} \frac{\partial F_1}{\partial x_1}(\mathbf{x}_0) & \cdots & \frac{\partial F_1}{\partial x_n}(\mathbf{x}_0) \\ \cdots & \cdots & \cdots \\ \frac{\partial F_m}{\partial x_1}(\mathbf{x}_0) & \cdots & \frac{\partial F_m}{\partial x_n}(\mathbf{x}_0) \end{bmatrix}$$

Differential: den lineære afbildning $dF(\mathbf{x}_0) : \mathbf{R}^n \to \mathbf{R}^m$,
$dF(\mathbf{x}_0)(\mathbf{h}) = DF(\mathbf{x}_0)\mathbf{h}$.
Affin approksimation: $G(\mathbf{x}_0 + \mathbf{h}) = F(\mathbf{x}_0) + DF(\mathbf{x}_0)\mathbf{h}$.
Rest: $R(\mathbf{h}) = F(\mathbf{x}_0 + \mathbf{h}) - G(\mathbf{x}_0 + \mathbf{h}) =$
$F(\mathbf{x}_0 + \mathbf{h}) - F(\mathbf{x}_0) - DF(\mathbf{x}_0)\mathbf{h}$.

Definition

F kaldes differentiabel i $\mathbf{x}_0$ hvis $\lim_{\mathbf{h} \to 0} \frac{R(\mathbf{h})}{\|h\|} = \mathbf{0}$.

C^1-funktioner er differentiable

$A = DF(\mathbf{x}_0)$ er den eneste 1. ordens approksimation

Theorem

En C^1 vektorfunktion $F : O \subseteq \mathbf{R}^n \to \mathbf{R}^m$ er

1. *kontinuert og*
2. *differentiabel.*

Theorem

Givet en afbildning $F : O \subseteq \mathbf{R}^n \to \mathbf{R}^m$ og et punkt $\mathbf{x}_0$.
Hvis der findes en $m \times n$ matrix A således at $F(\mathbf{x}_0 + \mathbf{h})$ og den affine afbildning $G(\mathbf{x}_0 + \mathbf{h}) = F(\mathbf{x}_0) + A\mathbf{h}$ har 1. ordens kontakt, så har F partielle afledede i $\mathbf{x}_0$ på formen $\frac{\partial F_i}{\partial x_j}(\mathbf{x}_0) = a_{ij}$,
dvs. $A = DF(\mathbf{x}_0)$.