# Bump Functions and Partitions of Unity

**When? Thu, September 23**
**8:45 – 11:45**
**Where? FrB7G5-109**

## Lectures

### Aims and Content

How can one paste locally given functions (sections etc.) together to yield a globally given function – with properties aimed for? The tool kit for this purpose consists of

- smooth bump functions that are zero outside a certain range. Remark that such functions can not be analytic, i.e., given by a convergent power series.
- Bump functions are collected in a partition of unity on the manifold. Using a partition of unity it is possible to average a sample of locally given functions, sections etc. on the manifold.

The main focus is not on the actual construction but on the certificate that functions, sections etc. with certain properties do exist. This will be used in several of the following lectures.

As preparation for the afternoon lectures, we define vector fields on $\mathbf{R}^n$ and study their action as derivation on function space. Vector fields on manifolds and their integral curves will be studied in the sequel.

**Lecturer:**

Rafael Wisniewski

**References:**

[LWT] ch. 13 and ch. 2.4-5.

**Exercises:**

- **LWT, ch. 12, p. 126:**
  Problem 12.2
- **LWT, ch. 13, p. 130:**
  Exercise 13.2

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# Vector Fields

**Thu, September 23; 12:30 – 15:30**

## Lectures

### Aims and Content

We will introduce the notion of a smooth vector field on a manifold. This is a map which assigns (in a smooth way) a tangent vector to each point of the manifold. It should be viewed as the analogue of a differential equation.
A curve whose velocity at each point is exactly equal to the given vector field is called an integral curve. The collection of all integral curves, called the flow of a vector field, gives a description of the motions on a manifold obeying to, e.g., the laws of mechanical physics. For instance, the possible motions of a spacecraft are given by a flow on a Lie group.

We will conclude the lecture by introducing two operations on vector fields that will be important during the last part og the course: Lie brackets and push-forwards of vector fields and their properties.

**Lecturer:**

Martin Raussen

**References:**

[LWT], ch. 14.

**Exercises:**

- **LWT, ch. 14, p. 144-146:**
  14.2, 14.1, 14.9, 14.10, 14.11, 14.13.

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