

Opgaver til 2. lektion

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$$\begin{aligned} 1) \quad (1) \quad EX &= 0.2 \\ \text{Var } X &= 0.4(-1-0.2)^2 + 0.6(1-0.2)^2 = 0.96 \end{aligned}$$

$$(2) \quad EX = 4$$

$$\begin{aligned} \text{Var } X &= \frac{1}{8}(1-4)^2 + \frac{1}{8}(2-4)^2 + \frac{1}{8}(3-4)^2 + \frac{1}{8}(4-4)^2 + \\ &\quad \frac{1}{4}(5-4)^2 + \frac{1}{4}(6-4)^2 = 3 \end{aligned}$$

$$(6) \quad X \sim b(3, 0.25)$$

$$EX = np = 3 \cdot 0.25 = 0.75$$

$$\text{Var } X = np(1-p) = 3 \cdot 0.25 \cdot 0.75 = 0.5625$$

$$(7) \quad X \sim b(40, 0.10)$$

$$EX = 40 \cdot 0.10 = 4$$

$$\text{Var } X = 40 \cdot 0.1 \cdot 0.9 = 3.6$$

$$2) \quad 2.1) \quad \bar{x} = 2.13 \quad s^2 = 0.98$$

$$2.2) \quad 2.13 \pm 1.96 \frac{\sqrt{0.98}}{\sqrt{10}} = [1.52; 2.74]$$

$$\left(\bar{x} \pm 1.96 \frac{s}{\sqrt{n}} \right)$$

$$2.3) \quad \text{Brede af konfidensinterval} \quad 2 \cdot 1.96 \frac{\sqrt{0.98}}{\sqrt{n}}$$

$$\text{Ønsket:} \quad 0.5 = 2 \cdot 1.96 \frac{\sqrt{0.98}}{\sqrt{n}} \Leftrightarrow$$

$$\sqrt{n} = \frac{1}{0.5} 2 \cdot 1.96 \cdot \sqrt{0.98} \Leftrightarrow$$

$$n = (7.76)^2 = 60.23$$

Dvs $n \geq 61$.

$$3) \quad 3.1) \quad P(X \leq 3) = 0.5 \quad \text{når } X \text{ normalfordelt} \\ \text{med middelværdi } 3.$$

$$3.2) \quad \mu \pm 1.96 \sigma \quad \text{dvs} \quad 3 \pm 1.96 \sqrt{2}.$$

$$4) \quad 4.1) \quad \bar{X} \approx N\left(3, \frac{2}{50}\right)$$

3

$$95\% \text{ interval: } \mu \pm 1.96 \sqrt{\frac{\sigma^2}{n}}$$

$$3 \pm 1.96 \sqrt{\frac{2}{50}}$$

$$4.2) \quad \bar{X} \pm 1.96 \sqrt{\frac{2}{50}}$$

$$5) \quad 5.1) \quad X = (X_1 + X_2 + \dots + X_{100}) \quad X_i \sim b(1, 0.25)$$

$$E X_i = p \quad \text{Var } X_i = p(1-p)$$

$$\bar{X} \approx N\left(p, \frac{p(1-p)}{100}\right)$$

$$\text{Med } p = 0.25 \quad \text{für } \frac{p(1-p)}{100} = 0.001875$$

$$P(\bar{X} \leq 0.1) \approx \text{pnorm}(0.1, 0.25, \sqrt{0.001875}) = 0.00027.$$

$$5.2) \quad \bar{X} = 0.65 \quad \text{Var } \bar{X} \text{ estimator ist } \frac{0.65(1-0.65)}{100}$$

95% Konfidenzintervall:

$$\bar{X} \pm 1.96 \sqrt{\frac{0.65(1-0.65)}{100}} = [0.55, 0.74]$$