Supppose $T_1, \dots, T_n$ are independent exponentially distributed with rate r. Then $\sum_i T_i$ is $\Gamma(n, r)$. Looking up the mean and variance for an inverse gamma distribution we obtain

$$\mathbb{E}\frac{1}{\sum_i T_i} = \frac{r}{n-1} \quad \mathbb{V}\text{ar}\frac{1}{\sum_i T_i} = \frac{r^2}{(n-1)^2(n-2)}$$

With $r = \lambda + \theta$ we obtain by independence

$$\mathbb{E}\frac{\sum_i \delta_i}{\sum_i T_i} = n\frac{\lambda}{\lambda+\theta}\frac{\lambda+\theta}{n-1} = \lambda\frac{n}{n-1}$$

Regarding the variance,

$$\begin{aligned}&\mathbb{V}\text{ar}\frac{\sum_i \delta_i}{\sum_i T_i}\\ =&\mathbb{V}\text{ar}\mathbb{E}[\frac{\sum_i \delta_i}{\sum_i T_i}|\sum_i \delta_i] + \mathbb{E}\mathbb{V}\text{ar}[\frac{\sum_i \delta_i}{\sum_i T_i}|\sum_i \delta_i]\\ =&\mathbb{V}\text{ar}[\sum_i \delta_i\frac{\lambda+\theta}{n-1}] + \mathbb{E}(\sum_i \delta_i)^2\frac{(\lambda+\theta)^2}{(n-1)^2(n-2)}\end{aligned}$$

This reduces to

$$\frac{n\lambda(\theta-\lambda+n\lambda)}{(n-1)^2(n-2)} + \frac{n\lambda\theta}{(n-1)^2}$$

This implies that the variance of $\sqrt{n}\frac{\sum_i \delta_i}{\sum_i T_i}$ converges to $\lambda(\lambda+\theta)$. This coincides with the asymptotic variance obtained by applying the delta-method to $g(\frac{1}{n}\sum_i \delta_i, \frac{1}{n}\sum_i T_i)$ with $g(x_1, x_2) = x_1/x_2$.