Exercises MM6 - Monte Carlo methods

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Suppose $U \sim N(0,1)$ and $Y|U = u \sim \text{Poisson}(\exp(\beta + U))$ (recall that $\text{Poisson}(\lambda)$ has density $f(y;\lambda) = \exp(-\lambda)\lambda^y/y!)$. Assume that $Y = 8$ is observed.

1) Compute and plot a simple Monte Carlo approximation of the likelihood and compute an estimate of the Monte Carlo error. Compare results obtained with different numbers of simulations. Consider also the situation where 10 observations 8, 18, 5, 7, 10, 9, 9, 6, 7, 10 are available (i.e. $f(8;\exp(\beta+u))$ is replaced by the product

$$\prod_{i=1}^{10}[f(y_i;\exp(\beta+u))$$

of conditional densities for these observations).

2 This exercise considers importance sampling approximation of the likelihood using a t-distribution with mean and variance from the Laplace approximation obtained in the exercises from MM5.

a) In the same plot draw the density of the importance t-distribution and of the joint density

$$\exp(g(u)) = \prod_{i=1}^{10}[f(y_i;\exp(\beta+u))/f(y_i;\bar{y})]f(u)$$

of the observations and the random effect (here we have for numerical stability divided with $f(y_i; \bar{y})$). How does the importance sampling distribution depend on the value of β ?
b) Compute an importance sampling approximation of the likelihood. What happens if you use the importance sampling t-distribution obtained for, say $\beta = 4$ to compute the likelihood for all the other values of β ?

3 Compute a parametric bootstrap p-value for the hypothesis of no subject specific random variation for the disruption data example.