The maximal degree in a Poisson-Delaunay graph

Gilles Bonnet *Ruhr-Universität Bochum, Germany*, E-mail: gilles.bonnet@ruhr-uni-bochum.de
Nicolas Chenavier *Université du Littoral Côte d'Opale, France*, E-mail: nicolas.chenavier@univ-littoral.fr

Keywords: Stochastic Geometry, Extreme Value Theory, Poisson-Delaunay graph, degree

Abstract: A Delaunay triangulation associated with a locally finite subset χ in $\mathbf{R}^2$ is a triangulation $DT(\chi)$ such that no point in χ belongs to the interior of the circumdisk of any triangle in $DT(\chi)$. This model is the key ingredient of the first algorithm for computing the minimal spanning tree and is extensively used in various domains, such as medical image segmentation and finite element methods to build meshes.

In this talk, we consider the case where $\chi = \eta$ is a homogeneous Poisson point process in $\mathbf{R}^2$. We investigate the maximal degree Δ_n of the so-called Delaunay graph associated with η (consisting of the set of edges of triangles) observed in the window $W_n = [-n, n]^2$, namely

$$\Delta_n = \max_{x \in \eta \cap W_n} d_\eta(x),$$

where $d_\eta(x)$ is the degree of any point x, i.e. the number of edges passing through x. As n goes to infinity, we show that Δ_n is concentrated on two consecutive integers. We also provide the exact order of Δ_n.