The maximal degree in a Poisson-Delaunay graph

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Abstract: A Delaunay triangulation associated with a locally finite subset $\chi$ in $\mathbb{R}^2$ is a triangulation $DT(\chi)$ such that no point in $\chi$ belongs to the interior of the circumdisk of any triangle in $DT(\chi)$. This model is the key ingredient of the first algorithm for computing the minimal spanning tree and is extensively used in various domains, such as medical image segmentation and finite element methods to build meshes.

In this talk, we consider the case where $\chi = \eta$ is a homogeneous Poisson point process in $\mathbb{R}^2$. We investigate the maximal degree $\Delta_n$ of the so-called Delaunay graph associated with $\eta$ (consisting of the set of edges of triangles) observed in the window $W_n = [-n,n]^2$, namely

$$\Delta_n = \max_{x \in \eta \cap W_n} d_\eta(x),$$

where $d_\eta(x)$ is the degree of any point $x$, i.e. the number of edges passing through $x$. As $n$ goes to infinity, we show that $\Delta_n$ is concentrated on two consecutive integers. We also provide the exact order of $\Delta_n$. 