L^p-boundedness of wave operators for two dimensional Schrödinger operators with threshold singularities

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Abstract

Let $H = H_0 + V$, $H_0 = -\Delta$ be the Schrödinger operator on $\mathbb{R}^2$ with very short range potential $|V(x)| \le C\langle x\rangle^{-\mathfrak{s}}$, $\mathfrak{s} > 2$. Then, wave operators for the pair (H, H_0) defined by the strong limits

$$W_\pm = \lim_{t\to\pm\infty} e^{itH}e^{-itH_0}$$

exist and we ask if they are bounded in $L^p(\mathbb{R}^2)$. We define

$$\mathcal{N}_\infty = \{u \in L^\infty(\mathbb{R}^2) \colon (-\Delta + V(x))u(x) = 0\}.$$

Then $u \in \mathcal{N}_\infty$ satisfies for some constants c, b_1, b_2 and $\varepsilon > 0$ that

$$u(x) = c + \frac{b_1x_1 + b_2x_2}{|x|^2} + O(|x|^{-1-\varepsilon}), \quad (|x| \to \infty)$$

and $u \in \mathcal{N}_\infty \setminus \{0\}$ is called *s-wave resonance* if $c \ne 0$, *p-wave resonance* if $c = 0$ but $(b_1, b_2) \ne (0, 0)$ and a zero energy eigenfunction of H if $c = b_1 = b_2 = 0$.

We prove the following theorem which generalizes results in Jensen-Yajima 2009 and Erdoğan-Goldberg-Green 2018.

Theorem 0.1. *Suppose $\langle x\rangle^2 V \in L^{\frac{4}{3}}(\mathbb{R}^2)$ and $\langle x\rangle^\gamma |V(x)| \in L^1(\mathbb{R}^2)$ for a constant $\gamma > 8$. Then, $W_\pm$ are bounded in $L^p(\mathbb{R}^2)$ for $1 < p < \infty$ if and only if H has no p-wave resonances. $W_\pm$ are otherwise bounded in $L^p(\mathbb{R}^2)$ for $1 < p \le 2$ and unbounded for $2 < p < \infty$.*

The proof uses the stationary representation of wave operators, Jensen-Nenciu's theory for the threshold analysis and various estimates of harmonic analysis.