IN THE WORLD ARE WE?

Version 2

A technical guide to datums and projections in New Zealand





newzealand.govt.nz



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In co-operation with the South Australian Spatial Information Committee.

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SYMBOL	DEFINITION
a	semi-major axis of reference ellipsoid
b	semi-minor axis of reference ellipsoid
<i>e</i> ²	squared eccentricity of reference ellipsoid
<i>E</i> , <i>N</i>	Easting and Northing
f	ellipsoid flattening
v	radius of curvature of the ellipsoid in the prime vertical
ϕ	latitude
λ	longitude
h	ellipsoidal height
H	normal-orthometric height in a vertical datum
N	geoid value
0	offset for a vertical datum
$\boldsymbol{R}_{x}, \boldsymbol{R}_{y}, \boldsymbol{R}_{z}$	coordinate transformation rotation parameters
$T_{\rm X}, T_{\rm Y}, T_{\rm Z}$	coordinate transformation translation parameters
<i>X</i> , <i>Y</i> , <i>Z</i>	Cartesian coordinates in a geocentric coordinate reference system
Δ_s	coordinate transformation scale change parameter.

SYMBOLS USED IN THIS BOOKLET

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WHERE IN THE WORLD ARE WE?

Since the dawn of civilisation, people have needed to measure and map their domain. Examples abound throughout history. Ancient Egyptians mapped land holdings in the valley of the Nile. Christopher Columbus, Ferdinand Magellan and others recorded their journeys of exploration. Many nations have recorded topographic information for military purposes.

Measuring and mapping continues today. The management of the world's natural and economic resources has become increasingly dependent on the availability of accurate and consistent geographic information. The methods for storing this data have changed radically in recent years, with paper maps giving way to computer-based storage, and manual drafting to digital production techniques. However, the underlying principles for ensuring spatial compatibility and consistency among data remain the same.

The foundation of any geographically based dataset is a spatial reference system. It is the mechanism through which grids can be placed on maps and navigation reliably achieved. A spatial reference system allows us to unambiguously identify locations through a set of coordinates (usually latitude and longitude or Northing and Easting) and to reliably calculate distances and areas.

This booklet aims to give a brief introduction to the development of spatial reference systems. It is intended for casual users of maps and navigation devices who want to know more about the coordinates that they use. It addresses some of the technical issues surrounding spatial reference systems, presenting formulae when appropriate. It also discusses the benefits of a spatial reference system to the community in general.

Every attempt has been made to present the material in non-technical terms. However, some reference to the science and terminology of geodesy and geodetic datums has been both inevitable and necessary. Hopefully, the associated explanations are sufficiently clear to the reader.

BACKGROUND

The need for a reference model

The shape of the Earth is very complex. Its surface includes plains, valleys, mountain ranges and deep oceans. In order to map its geography, we need a reference model that will allow such topographic irregularities to be recorded. The model needs to be simple so that it is easy to use. Also, the model needs to:

- include a coordinate system that allows the positions of features to be uniquely identified, and
- be readily associated with the physical world so that its use is intuitive.

A 'flat Earth' model

If the area being mapped is small (for example, 10 km square), a suitable reference model can be provided by a simple three-dimensional (3D) framework (see Diagram 1). The coordinate axes of the framework are arranged so that:

- the horizontal axes (N and E) are aligned in the directions of North and East, and
- the height axis (H) is perpendicular to the horizontal plane and is usually set coincident with sea level.



Diagram 1 – Local flat Earth 3D coordinate model

The positions of individual features on the Earth's surface are projected vertically onto the horizontal plane, allowing their positions relative to each other to be determined mathematically. Furthermore, the height of each feature relative to sea level is given by the vertical distance above or below the horizontal plane.

The orientation to North (the direction to the pole), together with the adoption of sea level as the reference for heights, provide the necessary association with the real world to make the reference system intuitively useable.

A model for the real (curved) Earth

Unfortunately, the simple 'flat Earth' model is unable to deal with the effects of the curvature of the Earth, which become significant when larger areas are to be measured or mapped.

There are two complicating factors:

- the curvature of the Earth, and
- the composition of the Earth's interior.

Curvature of the Earth

The curvature of the Earth forces the replacement of the 'flat Earth' model with a 'curved Earth' model. The selection of a curved reference surface is an important consideration as it needs to satisfy two criteria. Not only must it closely represent the shape of the Earth, but it must also be mathematically simple to use.

At first glance, the most appropriate figure would seem to be a sphere. It is geometrically very simple, but

Appearances can be deceptive!

In fact, because the Earth's equatorial diameter is significantly greater than the distance between the poles, a sphere is not the best choice for a reference surface.

An ellipsoid (also referred to as a spheroid) provides a better option. An ellipsoid is the figure generated by rotating an ellipse about its minor (shorter) axis (see Diagram 2). It has the advantages of accommodating the bulge at the Equator while remaining relatively simple mathematically. For these reasons, an ellipsoid is the figure usually chosen to represent the shape of the Earth.

If we ignore all land masses and imagine that the Earth is completely covered by water, then we would hope that the sea would form an absolutely smooth surface. An ellipsoid could be selected to match exactly the surface of the sea and become a perfect reference model.

Unfortunately there is a further complication!

Diagram 2 – The ellipsoid





Composition of the Earth's interior

The composition of the Earth is not uniform. It varies from place to place. There are variations in the density and distribution of the different rock types, and also irregularities caused by mountain ranges and ocean trenches. Together these lead to variations in the Earth's gravity field.

The surface that has a constant value of gravity is known as the geoid. You can't see the geoid but it's there. However, you can see the sea surface which coincides with the geoid to within one to two metres. The difference is caused by sea currents and variations in water temperature and density.

We saw earlier how, with the small-area 'flat Earth' model, the horizontal plane containing the N and E axes was able to be positioned so that it coincided with sea level. It is not possible to do this with a 'curved Earth' model due to the irregularity of the geoid and constantly changing sea surface. While it is possible to mathematically define the surface of the geoid, the model is very complex and not suitable for recording the geographic position of features. This problem is overcome by defining an ellipsoid that provides a best fit to the geoid (see Diagram 3).

Diagram 3 – Relationship between the ellipsoid and geoid



Implications for users

What are the implications of the lack of coincidence between the ellipsoid and the geoid?

The deflection of the vertical

First of all, there is the difference between the vertical or local gravity vector (the line perpendicular to the geoid/ sea level surface) and the ellipsoidal normal (the line perpendicular to the ellipsoid) (see Diagram 4).

The vertical is coincident with the direction of gravity at a point. It is the line along which an object falls when it is dropped. The vertical is very important to measurements taken by conventional surveying instruments (such as theodolites and levels). These instruments are set up so that their rotation axes are either coincident with, or perpendicular to, the vertical. Consequently, all angles are measured relative to the vertical.

The ellipsoidal normal, on the other hand, is the line along which a feature on the Earth's surface is projected down to the centre of the ellipsoid. It is the line used in computations involving observations at the feature.

To summarise:

- · the vertical is the line associated with measurements, while
- the ellipsoidal normal is the line associated with computations.

In our 'flat Earth' model, the two lines are considered coincident. In a 'curved Earth' model, they generally are not. Accordingly, to use an angle measurement in a computation process, the measurement should first be corrected for the 'difference' between the two lines. This difference is referred to as the deflection of the vertical (also referred to as deviation of the vertical). It is described by two small angles, the northerly and easterly components.

Convergence of the ellipsoidal normals

In our 'flat Earth' model, the lines projecting surface features to the horizontal plane were parallel. In a 'curved Earth' model, the ellipsoidal normals converge towards the centre of the ellipsoid (see Diagram 4).

Consequently, a distance measured on the surface of the Earth must be shortened, or in some cases lengthened, before it can be used in computations on the ellipsoid. The amount of the shortening will depend on the height of the measurement (above or below the ellipsoid). It is approximately one millimetre per kilometre for every 6.3 metres of height.

Diagram 4 – The vertical and the ellipsoidal normal



Height differences

Three reference surfaces are commonly used as a basis for height values. They are the sea level surface, the geoid, and the ellipsoid. In our 'flat Earth' model, the surfaces are considered coincident. This is rarely the case in a 'curved Earth' model.

In most parts of the world, distance above sea level has been the traditional mechanism for measuring height. This has been due to:

- a preference for a physically identifiable surface as a reference, and •
- the importance of sea level to economic activity (for example drainage on flood plains).

The geoid is a surface defined across the entire globe by gravity. All points that lie above or below the geoid have gravity values that are lower or higher than the geoid respectively.

Ellipsoidal heights (heights relative to the ellipsoid) are now becoming increasingly popular. Until recently they were more difficult to determine than sea level heights. However, the use of satellite-based positioning systems (eg GPS, GLONASS and Galileo) has reversed this situation because they give ellipsoidal heights as part of their positions.

The distance between the geoid and the ellipsoid is referred to as the geoid height (see Diagram 3). If a sea level height is known, it (or a height in terms of a vertical datum) can be converted to an ellipsoidal height, and vice versa.

The computation of the geoid and geoid-derived heights often uses gravity observations. The process is mathematically complex and beyond the scope of this discussion. Luckily there are pre-computed geoids available for use when converting heights. These are discussed later.

DATUMS AND PROJECTIONS

Geodesy

It is now time to define the term 'geodesy'. Essentially geodesy is the branch of science concerned with the determination of the size and shape of the Earth. Its range of contributing activities is vast. They include the acquiring and processing of survey measurements on (and above) the curved surface of the Earth, establishing geodetic datums and projections, and analysing gravity measurements.

Geodetic datum

A geodetic datum is a curved reference surface used to express the positions of features consistently. We define a geodetic datum by specifying a reference ellipsoid, the position (latitude and longitude) of an initial station and an azimuth from that station. A geocentric datum is a special case where the centre of the ellipsoid is defined as the centre of mass of the Earth. It is the simplified mathematical representation of the size and shape of the Earth.

A geodetic datum is vital to all activities involving spatial data. The ellipsoid provides a mathematical surface for performing surveying and navigation computations over a wide area. It is also a reference surface on which to base mapping and geographic information systems (GIS).

The ellipsoid is positioned so that it is a best fit to the Earth's geoid. An exact fit to the geoid is not possible due to undulations in the geoid caused by the variations in the Earth's gravity.

Mean sea level is widely used as the reference surface for the measurement of height. The contours on a map will usually show height above mean sea level. However, heights in terms of a geodetic datum will be in relation to the ellipsoid.

The current official geodetic datum in New Zealand is the New Zealand Geodetic Datum 2000 (NZGD2000).

Types of geodetic datum

Geodetic datums are usually classified into two categories. These are known as local geodetic datums and geocentric datums.

A local geodetic datum best approximates the size and shape of a particular part of the Earth's sea level surface. Invariably the centre of its ellipsoid will not coincide with the Earth's centre of mass.

Until very recently, most countries' survey information systems were expressed in terms of a local geodetic datum.

New Zealand Geodetic Datum 1949 (NZGD49) is an example of a local datum. Its ellipsoid is a good approximation to the size and shape of sea level surface in the region of New Zealand, but a poor approximation in other parts of the world (see Diagram 5).

Diagram 5 – Local geodetic datum



A geocentric datum best approximates the size and shape of the Earth as a whole. The centre of its ellipsoid coincides with the Earth's centre of mass (see Diagram 6). Geocentric datums do not seek to be a good approximation to any single part of the Earth, but on average they are a good fit.

Global Navigation Satellite Systems (GNSS), such as the Global Positioning System (GPS) operated by the United States Department of Defense, use geocentric datums to express their positions because of their global extent. The Russian GLONASS satellite navigation system also uses a geocentric datum. However, it is a different datum to that used by GPS.

The World Geodetic System 1984 (WGS84) and New Zealand Geodetic Datum 2000 (NZGD2000) are examples of geocentric datums.



Diagram 6 – Geocentric geodetic datum

Identifying position on a geodetic datum

Geodetic datum coordinates can be expressed in two forms: geographic and Cartesian.

The geographic coordinate system describes positions on the Earth's surface in terms of latitude, longitude and ellipsoidal height (see Diagrams 7, 8 and 9).

A set of geographical system coordinates is a mix of angular (latitude and longitude) and linear (height) values. The units of the angular components can be degrees, minutes and seconds, or degrees and decimals of degrees.

Diagram 7 – Geodetic latitude



Diagram 8 – Geodetic longitude



Diagram 9 – Ellipsoidal height



Cartesian coordinates are used to describe positions on the Earth's surface in terms of a three axis X, Y, Z system (see Diagram 10) where:

- the positive X axis lies in the Equatorial plane and passes through 0° longitude •
- the positive Y axis lies in the Equatorial plane and passes through 90° East longitude
- the positive Z axis is parallel to the Earth's rotation axis and passes through 90° North latitude.

Unlike the geographic system, all components in a set of Cartesian coordinates have the same linear units. This makes Cartesian coordinates easier to manipulate mathematically than geographicals. However, they are less intuitive for the user.

Diagram 11 illustrates the geodetic and Cartesian coordinates for a position P on the surface of the Earth.



Diagram 10 - Cartesian coordinate system





Converting coordinates: geographic to/from Cartesian

Geographic (latitude, longitude, ellipsoidal height) coordinates can be converted to Cartesian X, Y, Z values on the same geodetic datum using the formulae in Inserts 1 and 2.

INSERT 1 Conversion – Geographic to Cartesian

The formulae for converting latitude, longitude and ellipsoidal height to X, Y, Z are:

 $X = (v+h)\cos\phi\cos\lambda$ $Y = (v+h)\cos\phi\sin\lambda$ $Z = \left[v \left(1 - e^2 \right) + h \right] \sin \phi$

where

$$v = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$$
$$f = \frac{1}{f^{-1}}$$
$$e^2 = 2f - f^2$$

and

X, Y, Z	are the Cartesian coordinates of the point
ϕ, λ, h	are the latitude, longitude and ellipsoidal height of the point
a	is the length of the semi-major axis of the reference ellipsoid
<i>e</i> ²	is the squared eccentricity of the reference ellipsoid
f	is the ellipsoid flattening, and
ν	is the radius of ellipsoid in the prime vertical.

INSERT 2 Conversion – Cartesian to Geographic

The formulae for converting X, Y, Z to latitude, longitude and ellipsoidal height are:

$$\tan \lambda = \frac{Y}{X}$$
$$\tan \phi = \frac{Z(1-f) + e^2 a \sin^3 \mu}{(1-f)(p - e^2 a \cos^3 \mu)}$$
$$h = p \cos \phi + Z \sin \phi - a \sqrt{1-e^2 \sin^2 \phi}$$

where

$$p = \sqrt{X^2 + Y^2}$$
$$r = \sqrt{p^2 + Z^2}$$
$$\tan \mu = \frac{Z}{p} \left[(1 - f) + \frac{e^2 a}{r} \right]$$

X, Y, Z	are the Cartesian coordinates of the point
<i>φ</i> , <i>λ</i> , <i>h</i>	are the latitude, longitude and ellipsoidal height of the point
а	is the length of the semi-major axis of the reference ellipsoid
e^2	is the squared eccentricity of the reference ellipsoid, and
f	is ellipsoid flattening.

A multitude of datums

Ideally, we would have just one geodetic datum so that there is one unambiguous way to assign ellipsoidal coordinates to points. But as we are able to measure positions more accurately, and as continents move, a number of different datums have been developed and continue to be developed.

In practice it is frequently necessary to deal with two or more datums for technical or political reasons (for example, a local datum for existing mapping and a newer geocentric datum for satellite navigation).

It is important to understand that the coordinate values for a point are dependent on the datum being used. The latitude, longitude and height of a point defined on Datum 1 will almost certainly be different to its latitude, longitude and height defined on Datum 2. The differences may be a consequence of:

- the ellipsoids being different shapes
- the centres of the ellipsoids being displaced, possibly by hundreds of metres, or
- the Cartesian coordinate axes of the two datums not being parallel or being subject to a scale difference.

Datums may also be classified as:

• Static

The datum is defined by the coordinates of key stations which are held fixed. New Zealand Geodetic Datum 1949 was a static datum.

• Semi-dynamic

The datum is defined by its relationship to a dynamic global reference frame at a specified time (reference epoch). The datum definition is fixed at this time and does not include time dependencies. Modelling of uniform time dependencies, such as the effect of crustal deformation due to plate tectonics, may be applied during calculations to remove their effect. Coordinates at the reference epoch may change due to the acceptance of new survey data, earthquakes or localised mark movement. New Zealand Geodetic Datum 2000 is a semi-dynamic datum.

• Dynamic

The datum is defined continuously by its relationship to a dynamic global reference system such as the International Terrestrial Reference System (ITRS).

It is vital for users to be aware of the datum(s) they are using. Directly combining coordinates from two different datums will produce wrong numbers and can result in catastrophic outcomes.

Transforming between datums

Coordinates can be converted from one datum to another if the relationship between the two is known. The relationship is described by two components:

- a set of formulae that describe the mathematics of the transformation process, and
- a set of parameters, referred to as transformation parameters, that are used in the formulae.

It is common for more than one transformation to be defined to convert between datums. Typically there will be simple low accuracy transformations, and more complex transformations to higher accuracy.

The transformation parameters are derived by analysing survey control stations with coordinates on both datums. The minimum number of stations required for this process depends on the transformation method being proposed. There is no maximum limit, and in general as many stations as possible are used.

The accuracy of a transformation can be assessed by comparing transformed coordinates at points that have coordinates defined in both datums.

Consider the typical situation where we have:

- · Coordinate Set 1 which has been computed from Network 1 on Datum 1, and
- Coordinate Set 2 which has been computed from Network 2 on Datum 2.

Generally, if the coordinates from Set 1 are converted to Datum 2, they will not exactly match the coordinates of Set 2. This is because the coordinate sets are derived from different measurement sets or network geometries. (For example, Network 1 may have been measured by triangulation and Network 2 by GPS.) One network will appear distorted in relation to the other.

The differences between the transformed Datum 1 coordinates and the network Datum 2 coordinates are known as residuals. Their magnitude provides an indication of the quality of the network held in the two datums, as well as an indication of the accuracy of the transformation between those datums.

Transformation parameters are commonly generated by government mapping organisations and are freely available to users.

Common transformation models

3-parameter transformation

If the *X*, *Y*, *Z* coordinate axes of two datums are assumed to be parallel and identically scaled, a 3-parameter transformation can be derived to represent their relationship (see Diagram 12 and Insert 3).

Diagram 12 – 3-parameter transformation



INSERT 3 3-parameter transformation formulae

Use the following formulae to convert Cartesian coordinates from Datum 1 to Datum 2 values with a 3-parameter transformation:

$$X_2 = X_1 + T_X$$
 $Y_2 = Y_1 + T_Y$ $Z_2 = Z_1 + T_Z$ where X_1, Y_1, Z_1 are the Cartesian coordinates of Datum 1 X_2, Y_2, Z_2 are the Cartesian coordinates of Datum 2, and T_X, T_Y, T_Z is the difference between the centres of the two ellipsoids in the sense of Datum 1 to Datum 2.

7-parameter transformation

If the coordinate axes are not parallel and identically scaled, a 7-parameter transformation can be derived (see Diagram 13 and Insert 4).

Diagram 13 – 7-parameter transformation



INSERT 4 7-parameter transformation formulae

Use the following formulae to convert Cartesian coordinates from Datum 1 to Datum 2 values with a 7-parameter transformation:

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} + \begin{bmatrix} 1 + \Delta_S \times 10^{-6} \end{bmatrix} \begin{bmatrix} 1 & +R_Z & -R_Y \\ -R_Z & 1 & +R_X \\ +R_Y & -R_X & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

where

- X_1, Y_1, Z_1 are the Cartesian coordinates of Datum 1
- X_2, Y_2, Z_2 are the Cartesian coordinates of Datum 2
- T_X, T_Y, T_Z is the difference between the centres of the two ellipsoids
- $R_{\chi}, R_{\chi}, R_{\chi}$ are the rotations around the three coordinate axes, and
- Δ_{s} is the scale difference between the coordinate systems.

Rotations are positive anticlockwise about the axes of Datum 2 coordinate system when viewing the origin from the positive axes.

Map projections

Another type of coordinate system is provided by map projections. A map projection enables the curved surface of the ellipsoid to be represented on a flat sheet of paper (in other words, a map). Projections are used to define local coordinate systems in cases where calculations of distance, direction, and area are less complex than the equivalent computations on the ellipsoid.

Many projections can be defined in terms of a particular geodetic datum, but each projection is linked to one geodetic datum.

The projection process results in the map's spatial representation being distorted. Imagine stretching and tearing a basketball to make its curved surface lie flat on the ground. The magnitude of the distortion can be calculated, allowing corrections to be made when necessary.

Every map projection uses a rectangular grid coordinate system (similar to our 'flat Earth' grid). Map projection coordinates are described in terms of Northing and Easting, being distances to the North and East of an origin. They are usually expressed in units of metres.

There are many types of map projections, each one representing a different way of distorting the surface of the ellipsoid into a plane. One of the most commonly used is the Transverse Mercator Projection.

The Universal Transverse Mercator (UTM) is a global implementation of the Transverse Mercator Projection. It divides the Earth into 60 zones, each bounded by meridians of longitude and extending from 84° N to 80° S (see Insert 5). Picture an orange containing 60 segments. Each segment would be equivalent to a UTM zone.

The meridian at the zone's centre is referred to as the Central Meridian. Each UTM zone has a width of six degrees of longitude, which avoids distortions that occur if the width becomes too large.

The formulae for converting geographic latitude and longitude to grid Easting and Northing differ for each type of projection and are too complex to be quoted in this document. However, they can be found in reference books on map projections, and are built into most mapping software.

INSERT 5

North

Universal Transverse Mercator (UTM)

The point of intersection between the Equator and the Central Meridian is assigned the following values, ensuring that all coordinates within the zone are greater than zero:

East 500,000.000 metres

0.000 metres (Northern Hemisphere) or 10,000,000.000 metres (Southern Hemisphere)

Under the UTM system, each East and North coordinate pair exists in each of the sixty zones. Consequently, the zone number must be quoted with the East and North values. The zone number is effectively a third coordinate.

New Zealand lies in UTM Zones 58, 59 and 60.

THE GEODETIC SYSTEM IN NEW ZEALAND

Geodetic networks

The **National Geodetic System** in New Zealand is defined in section 4 of the Cadastral Survey Act 2002 as "a system that enables positions on the surface of the Earth to be determined by reference to a mathematical model that describes the size and shape of the Earth".

The Cadastral Survey Act 2002 also defines the **National Survey Control System** as "a system used to determine the position of points, features and boundaries in cadastral surveys, other surveys, and land information systems".

The National Geodetic System consists of reference marks in the ground and a set of mathematical parameters and equations that allow the coordinates of these marks and other points to be determined. It provides the spatial referencing framework for the cadastral survey system and datum monitoring. It also enables the compatible positioning of spatial information such as topographic mapping and other land information.

In New Zealand, the positions of the reference marks are recorded in terms of the official geodetic datum (NZGD2000), which in turn is linked to a global reference system compatible with international positioning systems.

Traditionally, geodetic surveying involved taking surveying measurements such as angles, distances, height differences and astronomical observations, and then processing them to produce coordinates. More recently and for the development of NZGD2000, measurement systems based on artificial Earth satellites, termed Global Navigation Satellite Systems (GNSS) have become available. A number of systems operate, including GPS, GLONASS and Galileo.

The geodetic reference marks, often referred to as trig stations, normally consist of brass plaques or stainless steel pins set in concrete. They have been traditionally located on hill tops, and have frequently been marked by some form of survey beacon. The beacons both help to locate the stations and protect them from possible destruction. They also provide a landmark for distant users.

These marks are classified into six geodetic orders, from Zero Order (most accurate) to the Fifth Order (least accurate), based on the accuracy of the mark's coordinate.

The fundamental geodetic reference network of continuous GNSS tracking stations provides a connection from the official geodetic datum(s) to the applicable global reference frame(s) and monitors large-scale differential Earth movements around New Zealand at the regional tectonic plate scale. This is called the PositioNZ network (see Diagram 14) and enables users to position points to an accuracy of a few centimetres relative to the official datum. This network is made up of Zero Order marks.





The New Zealand Geodetic Datum 1949 (NZGD49)

The New Zealand Geodetic Datum 1949 (NZGD49) (see Insert 6) is a horizontal datum only, and does not define heights. It is a static datum because the coordinates of the trig stations defining the datum (First Order stations) have been held fixed since they were defined in 1949. It uses the International (Hayford) Spheroid, which was recommended to New Zealand by the International Association of Geodesy (IAG) for national datums. At that time, prior to satellite measurements, it was considered to be the best estimate of the shape of the Earth.

INSERT 6

New Zealand Geodetic Datum 1949

The origin station for the New Zealand Geodetic Datum 1949 is the assumed position of Papatahi, in the Rimutaka Range near Wellington. This position is based on latitude determinations at 60 stations, azimuth determinations at 22 stations, and the longitude of the Dominion Observatory at Kelburn, Wellington, as determined in the Second International Longitude Operation 1933. Papatahi's coordinate values are:

Geodetic latitude	41° 19' 08.9" South
Geodetic longitude	175° 02' 51.0" East
Azimuth to Kapiti No 2	347° 55' 02.5"
Reference ellipsoid	International (Hayford) Spheroid
Reference ellipsoid Semi-major axis (<i>a</i>)	International (Hayford) Spheroic 6 378 388.000 metres

New Zealand Geodetic Datum 2000 (NZGD2000)

In 1998, New Zealand adopted a new geocentric datum, New Zealand Geodetic Datum (NZGD2000), as its national datum (see Insert 7). This is a semi-dynamic datum with coordinates aligned to the International Terrestrial Reference Frame 1996 (ITRF96) at a reference epoch of 1 January 2000. It uses the Geodetic Reference System 1980 (GRS80) reference ellipsoid.

Positions in terms of NZGD2000 have latitude, longitude and ellipsoidal height elements. The adoption of NZGD2000 (ie the replacement of NZGD49) has allowed closer integration with international coordinate frameworks and navigation systems. In particular, NZGD2000 coincides almost exactly with WGS84, which is the datum supporting the Global Positioning System. This enables GPS-derived coordinates to be used directly with NZGD2000 in most circumstances.

INSERT 7

New Zealand Geodetic Datum 2000

Eleven global reference stations were held fixed when computing the coordinates for NZGD2000. The NZGD2000 implementation parameters are:

Reference ellipsoid	Geodetic Reference System 1980 (GRS80)
Semi-major axis (a)	6 378 137.000 metres
Flattening (f)	1/298.257222101
International Terrestrial	
Reference System (ITRS)	ITRF96
Reference epoch	1 January 2000 (2000.0)
Deformation model	The NZGD2000 Deformation Model (described below) to compute NZGD2000 coordinates

The NZGD2000 Deformation Model (Beavan and Haines, 2001) is used to account for broad-scale deformation across New Zealand, primarily due to the effects of plate tectonics. The current Deformation Model used in the definition of NZGD2000 assumes a constant velocity through time (see Diagram 15). It enables coordinates and observation to be transformed between the reference date of the datum (2000.0) and the observation date.

Diagram 15 – New Zealand Deformation Model



An Important Warning!

There are major differences between the NZGD49 and NZGD2000 coordinate systems.

Remember!

- New Zealand Geodetic Datum 1949 coordinates use a local datum, while the New Zealand Geodetic Datum 2000 coordinates use a geocentric datum
- · the two datums use different-shaped ellipsoids, and
- the centres of the two ellipsoids are offset one from another.

As a result, the NZGD49 and NZGD2000 coordinates for the same point differ by approximately 200 metres (approximately 10 metres in the East and 190 metres in the North).

The adoption of a new datum changes everything.

Converting coordinates between NZGD49 and NZGD2000

For low (± 15 metre) accuracy requirements the procedure for transforming coordinates from NZGD49 to NZGD2000 is:

- 1. If necessary, transform projection coordinates (Northing and Easting) on NZGD49 to geographic (latitude and longitude) coordinates on NZGD49 using the appropriate map projection formulae.
- 2. Transform the geographic coordinates on NZGD49 to geographic values on NZGD2000 using the corrections given in Insert 8.
- If necessary, convert the geodetic coordinates on NZGD2000 to projection coordinates on NZGD2000 using the appropriate map projection formulae.

INSERT 8

Transformation from NZGD49 to NZGD2000

To compute a low accuracy transformation, apply the nationwide mid-point value of the range of differences in latitude and longitude between NZGD49 and NZGD2000.

Adjust the NZGD49 latitude by 6.1 seconds Northwards and the longitude by 0.5 seconds Eastwards.

- Apply these adjustments in the opposite sense when transforming from NZGD2000 to NZGD49.
- This transformation will be accurate to within ± 15 metres.

For medium (± 4 - 5 metre) accuracy requirements two options are available to relate the NZGD49 coordinate set to the NZGD2000 datum (and vice versa).

The procedure for transforming coordinates from NZGD49 to NZGD2000 is:

- 1. If necessary, transform projection coordinates (Northing and Easting) on NZGD49 to geographic (latitude and longitude) coordinates on NZGD49 using the appropriate map projection formulae.
- 2. Convert the geographic coordinates on NZGD49 to *X*, *Y*, *Z* Cartesian values on NZGD49 using the formulae in Insert 1.
- 3. Transform the NZGD49 *X*, *Y*, *Z* Cartesian values to NZGD2000 *X*, *Y*, *Z* Cartesian values, using the parameters in either Insert 9 or 10.
- 4. Convert the *X*, *Y*, *Z* Cartesian values on NZGD2000 to geographic coordinates on NZGD2000 using the formulae in Insert 2.
- 5. If necessary, convert the geodetic coordinates on NZGD2000 to projection coordinates on NZGD2000 using the appropriate map projection formulae.

INSERT 9

Transformation from NZGD49 to NZGD2000 3-parameter transformation

The three parameters for transforming NZGD49 coordinates to NZGD2000 are:

 T_X +54.4 metres

- T_{γ} -20.1 metres
- T_Z +183.1 metres
- The parameters given here are contained in the Land Information New Zealand Standard 25000 NZGD2000.
- Due to distortions in the NZGD49 the transformation will be accurate to within ± 5 metres.
- To transform from NZGD2000 to NZGD49, reverse the signs on all parameters.

INSERT 10

Transformation from NZGD49 to NZGD2000 7-parameter transformation

The seven parameters for transforming NZGD49 coordinates to NZGD2000 are:

- T_X +59.47 metres
- T_{γ} -5.04 metres
- T_Z +187.44 metres
- R_X -0.470 seconds
- R_{γ} +0.100 seconds
- R_Z -1.024 seconds
- $\Delta_{\rm s}$ -4.5993 parts per million
- The parameters given here are contained in the Land Information New Zealand Standard 25000 NZGD2000.
- Due to distortions in the NZGD49, the transformation will be accurate to within ± 4 metres.
- To transform from NZGD2000 to NZGD49, reverse the signs on all parameters.
- The rotations must be converted to radians prior to inclusion in the formula.

For high (0.1 meter or better) accuracy requirements, a transformation file of coordinate shifts has been produced. This is available in the Geodetic Information section on the Land Information New Zealand website, **www.linz.govt.nz**. The transformation file allows the conversion of coordinates from NZGD49 to NZGD2000.

Use this procedure to convert coordinates from NZGD2000 to NZGD49:

- 1. Use the NZGD2000 coordinates in the grid to determine the NZGD49 to NZGD2000 correction.
- Reverse this correction and apply to the NZGD2000 coordinates to determine initial estimates of NZGD49 coordinates.
- Use the estimated NZGD49 coordinates to determine a more accurate NZGD49 to NZGD2000 correction.
- Reverse this correction and apply to the NZGD2000 coordinates to determine the final NZGD49 coordinates.

Vertical datum

Although modern geodetic datums include ellipsoidal heights to define positions, these heights are often unsuitable for many applications. This is because ellipsoidal heights relate to the ellipsoid and not the geoid and so, for example, they can not indicate the direction of water flow or even the level of the sea.

Vertical datums have been created historically by observing the level of the sea at a tide gauge, relating this to a mark on the ground (datum origin) and then transferring this height (by precise levelling) to distant locations. Heights established with this method are often referred to as orthometric heights. Because we don't have gravity observations at all bench marks in New Zealand, we use a gravity model to approximate the gravity field. This is called the normal-orthometric height system.

A geoid model can be used to convert heights between the more easily observed ellipsoidal heights and the more useful normal-orthometric heights. The relationship between the different heights is shown in Diagram 16.



Diagram 16 – The geoid, ellipsoid and terrain

In New Zealand, the normal-orthometric height system is generally used and the heights are related to one of 13 different vertical datums (Diagram 17). These datums give heights in relation to sea level at the datum origin. A complicating factor is that because the sea level is not constant around New Zealand, the different vertical datums are vertically offset from each other. Land Information New Zealand has calculated a geoid for New Zealand (NZ Geoid 2005, see Diagram 18) that can be used to transform ellipsoidal heights to sea level (orthometric) heights. The method for transforming heights between height systems is shown in Insert 11.





Diagram 18 – New Zealand Geoid 2005



INSERT 11

Ellipsoidal to normal-orthometric height conversion

Use the following formula to convert between ellipsoidal and normal-orthometric heights: H = h + N + o

where:

Н	is the normal-orthometric height in a vertical datum
h	is the NZGD2000 ellipsoidal height
Ν	is the NZ Geoid 2005 value (based on the position of the point) interpolated from the Land Information New Zealand (LINZ) website, and
0	is the offset between different vertical datums (refer to the LINZ website).

Transformations undertaken using this formula may be accurate to within 0.1 metres due to errors in both the geoid model and the datum offset.

Projections

The main projection systems used in New Zealand are:

- the Meridional Circuits (see Insert 12), a set of 28 local Transverse Mercator projections used in cadastral surveying to simplify calculations, and
- the New Zealand Transverse Mercator 2000 Projection (see Insert 13), which is used for the current 1:50,000 topographic map series (NZTopo50).

The previous map series (Topomap 260) used a different and unique type of projection called New Zealand Map Grid (NZMG) (see Insert 14). More details on the New Zealand map projections are available in the Geodetic Information section on the LINZ website, **www.linz.govt.nz**.

A technical guide to datums and projections in New Zealand

INSERT 12

New Zealand Meridional Circuits 2000

Twenty-eight new meridional circuits in terms of NZGD2000 were introduced in 1999 to replace the existing 28 circuits which were in terms of NZGD49. The parameters for the new circuits are:

Datum	NZGD2000
Projection	Transverse Mercator
The coordinates of the origin of each circuit are:	800,000 metres North
	400,000 metres East

This means that all eastings are always less than all northings to avoid confusion.

circuit name	origin latitude	origin longitude	Central Meridian
			scale factor
Mount Eden 2000	36° 52' 47" S	174° 45' 51" E	0.9999
Bay of Plenty 2000	37° 45' 40" S	176° 27' 58" E	1.0
Poverty Bay 2000	38° 37' 28" S	177° 53' 08'' E	1.0
Hawke's Bay 2000	39° 39' 03'' S	176° 40' 25" E	1.0
Taranaki 2000	39° 08' 08'' S	174° 13' 40" E	1.0
Tuhirangi 2000	39° 30' 44'' S	175° 38' 24'' E	1.0
Wanganui 2000	40° 14' 31'' S	175° 29' 17'' E	1.0
Wairarapa 2000	40° 55' 31" S	175° 38' 50" E	1.0
Wellington 2000	41° 18' 04'' S	174° 46' 35'' E	1.0
Collingwood 2000	40° 42' 53" S	172° 40' 19" E	1.0
Nelson 2000	41° 16' 28" S	173° 17' 57" E	1.0
Karamea 2000	41° 17' 23'' S	172° 06' 32'' E	1.0
Buller 2000	41° 48' 38" S	171° 34' 52" E	1.0
Grey 2000	42° 20' 01'' S	171° 32' 59" E	1.0
Amuri 2000	42° 41' 20" S	173° 00' 36" E	1.0
Marlborough 2000	41° 32' 40" S	173° 48' 07'' E	1.0
Hokitika 2000	42° 53' 10" S	170° 58' 47'' E	1.0
Okarito 2000	43° 06' 36" S	170° 15' 39" E	1.0
Jacksons Bay 2000	43° 58' 40" S	168° 36' 22'' E	1.0
Mount Pleasant 2000	43° 35' 26" S	172° 43' 37" E	1.0
Gawler 2000	43° 44' 55" S	171° 21' 38" E	1.0
Timaru 2000	44° 24' 07'' S	171° 03' 26" E	1.0
Lindis Peak 2000	44° 44' 06'' S	169° 28' 03'' E	1.0
Mount Nicholas 2000	45° 07' 58'' S	168° 23' 55" E	1.0
Mount York 2000	45° 33' 49" S	167° 44' 19" E	1.0
Observation Point 2000	45° 48' 58" S	170° 37' 42'' E	1.0
North Taieri 2000	45° 51' 41" S	170° 16' 57'' E	0.99996
Bluff 2000	46° 36' 00'' S	168° 20' 34" E	1.0

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INSERT 13

New Zealand Transverse Mercator 2000 (NZTM2000)

New Zealand Transverse Mercator 2000 (NZTM2000) replaced NZMG in 2000 as the projection for topographic and related maps on scales of 1:500 000 or greater:

Datum	NZGD2000	
The true origin is:		
Latitude	0° 00' 00''	South
Longitude	173° 00' 00''	East
The coordinates of this point are:	10,000,000	metres North
	1,600,000	metres East
Central Meridian scale factor:	0.9996	

This means that all eastings are always less than all northings to avoid confusion. They are also sufficiently different from NZMG coordinates to avoid confusion.

INSERT 14 New Zealand Map Grid (NZMG)

In 1973, a conformal mapping projection with minimum scale error, New Zealand Map Grid (NZMG), was adopted for plotting topographic and related maps on scales of 1:500 000 or greater. The range of scale enlargement is from +0.023% to -0.022%, considerably less than any other projection previously used for the whole of New Zealand. NZMG is used as the projection in the Topomap 260 series of topographic maps:

Datum NZGD49

The true origin is:

Latitude	41° 00' 00"	South
Longitude	173° 00' 00"	East
The coordinates of this point are:	2,510,000	metres East
	6,023,150	metres North

This results in all eastings always being less than 5,000,000 metres, and all northings greater than 5,000,000 metres, to avoid confusion between eastings and northings.

For further information on NZMG see Reilly, W. I., (1973): A conformal mapping projection with minimum scale error. *Survey Review, Vol 22, No 168.*

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THE VALUE OF A GEODETIC REFERENCE SYSTEM

(This chapter draws heavily on "The Use and Value of a Geodetic Reference System", published by Earl F. Epstein and Thomas D. Duchesneau at the University of Maine in 1984.)

A geodetic network, and its associated spatial reference system, is a fundamental component of a nation's infrastructure. Rather than being an end in itself, it derives its value from being an input to other production processes.

The utility of the network is determined by identifying the products that are dependent upon the network's unique properties. The unique property of the geodetic network is its ability to integrate multiple geographically dependent data sources into a single geographic reference frame.

The concept of a land information system (LIS) is illustrated in Diagram 19. Typically, the system will be multi-layered, each layer comprising data relating to a particular theme. For example, one layer may represent the roads in an area. A second layer may illustrate the distribution of a particular plant or animal. Further layers may contain the noise contours surrounding an aircraft flight path, or the location of electricity transmission lines.

Diagram 19 – Layers in a land information system



Modern land information systems are computer based. The data for these layers may be stored on a single computer or on the computers of the custodial authorities and accessed by networking or other means.

To function effectively, the LIS must possess one **essential** attribute. It must be able to geographically relate and inter-relate the data in its layers. For example, consider a LIS layer that represents the distribution of native vegetation in a region. The usefulness of the layer will be governed by how accurately it represents the position and size of one pocket of vegetation relative to other such pockets. A spatial referencing system that permits the definition of position and extent in terms of coordinates is more useful in this situation.

However, the adoption of such a system becomes even more important when the data in two or more layers are combined. To inter-relate the data from different themes you need to use a common coordinate system for both. For example, you might want to inter-relate your native vegetation data with data that

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describes the distribution of deer in the same region. If the positions of the deer concentrations and vegetation pockets were represented in different and independent coordinate systems, the inter-relation process would be very difficult. However, using the same geographic referencing system for both data sets means the required spatial relationships could be determined very quickly and efficiently.

Therefore, the spatial framework in an LIS acts as a medium through which data sets can be inter-related geographically. The geodetic network provides survey marks whose positions are accurately determined in terms of a single national coordinate system. If the positions of the items in each layer of the LIS are described with this system (either through direct measurement to the survey marks, or from the grid on a map), the data sets can be integrated efficiently.

Often, the secondary and tertiary users of information create the demand for universal compatibility of geographically related data. The authorities that generate data sets for a specific purpose (primary users) tend to have little interest in the needs of those who might want to inter-relate the data sets later. For example, organisations compiling data on soils and geology (for agriculture and mining) may not consider the needs of hydrologists who are interested in correlating that data with water run-off. The hydrologist, as a secondary or tertiary user of the data, clearly needs a way to make the data sets compatible.

In the absence of a common spatial framework, how else could a user inter-relate data sets? There are alternatives. For example, you could measure how the elements of each data set relate to easily identifiable physical features. This allows you to inter-relate through these common features. However, to use this process you would also need to take measurements between the physical features themselves. This is so that you can establish their relative positions in a local coordinate system. Doing this immediately raises questions regarding efficiency.

Because users or organisations would have to duplicate measurement processes to inter-relate data sets, the community incurs a clear economic cost as a result. Therefore, it follows that a common spatial framework benefits the community economically by avoiding the costs of duplication.

This is the crux of the argument on the value of a common spatial framework and geodetic network. If a common geographical referencing system were not available, we would need other ways to inter-relate data sets. A permanently monumented geodetic network effectively means that the bulk of the required measurements only ever have to be done once. Connecting the individual data sets to local control stations allows the positions of the data items to be expressed in a common coordinate system, avoiding the waste that results from separately inter-relating each pair of data sets. Clearly, the avoided costs would be considerable, effectively representing the costs of re-measuring the network several times over without gaining the benefit of universal consistency.

An investigation into the value of a geodetic network was undertaken in the United States of America (Epstein, E.F. and Duchesneau, T.D, "The Use and Value of a Geodetic Reference System", University of Maine at Orono, April 1984). The investigation aimed to quantify the avoided costs resulting from network availability through an analysis of case studies. The case studies included projects involving:

- land use and development plans
- · watershed and related water studies, and
- · construction of capital works, in particular highway construction.

Each of the cases was characterised by a frequent need for accurate and compatible data.

The study concluded that the ratio of benefits to costs flowing from the network lay in the range 1.7 to 4.5. Furthermore, the study authors considered these figures as conservative due to the non-availability of certain data relevant to the study.

A FINAL WORD

The common spatial framework provided by a geodetic datum and network is a major resource for the New Zealand community. The network provides the physical infrastructure through which New Zealand's geographical referencing system is established and maintained. This in turn allows the positions of all Earth-related information to be expressed in a common coordinate system, giving us large efficiencies when integrating dissimilar or spatially separated data.

The need to know location and position is so pervasive throughout the activities of New Zealand society, that it is almost impossible to fully appreciate the extent to which the geographic referencing system is used. Indeed the demand for universal compatibility among data sets by those who are not part of the survey industry is extensive and appears to be growing. An increasing number of projects will owe both their economic and technical viability to the spatial infrastructure defined by New Zealand's geodetic network.

ACKNOWLEDGMENTS

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The original paper was prepared from training material and presentations gathered or developed by Andrew Jones over a number of years.

The chapter entitled "The Value of a Geodetic Reference System" draws heavily on work published by Earl F. Epstein and Thomas D. Duchesneau at the University of Maine in 1984.

The formulae in Inserts 1 and 2 were taken from "GPS Theory and Practice" by B. Hofmann-Wellenhof, H. Lichtenegger, and J. Collins (Springer-Verlag, 1992).

Information regarding the definition of the New Zealand Geodetic Datum 1949 in Insert 4 was taken from "First-order geodetic triangulation of New Zealand, 1909-49 and 1973-74", Department of Lands and Survey Technical Series No. 1, by L.P. Lee.

Information regarding the definition of the New Zealand Map Grid in Insert 4 was taken from "Department of Lands and Survey Technical Circular 1973/32".

The transformation parameters in Inserts 9 and 10 are detailed in Land Information New Zealand Standard LINZS25000 NZGD2000.

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Beavan, J. and Haines, J. (2001). "Contemporary horizontal velocity and strain rate fields of the Pacific-Australian plate boundary zone through New Zealand". Journal of Geophysical Research, 106, B1, 741-770.





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