Discussion contribution to the paper 'Exact and computationally efficient likelihood-based estimation for discretely observed diffusion processes' by Beskos, Papaspiliopoulos, Roberts and Fearnhead

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I wonder how successfully the ideas of this interesting and stimulating paper can be extended to the case of a Cox process X_t, where V_t is a non-negative diffusion process and X_t conditional on V_t is a Poisson process with intensity function V_t. Suppose that $t_1, \ldots, t_n$ are the events of the Cox process observed on a finite time interval $[0, a]$ and, as in the paper, we only observe the diffussion process at the times $0 = t_0 < t_1, \ldots < t_n$. Using the notation in the paper, the likelihood is

$$L(\theta|\mathbf{v}, t_1, \ldots, t_n) = E_\theta \left[\exp\left(-\int_0^a V_t \,\mathrm{d}t \right) \prod_{i=1}^n V_{t_i} \times p_{\Delta t_i}(V_{t_{i-1}}, V_{t_i}; \theta) \,\middle|\, \mathbf{v} \right]$$

where the conditional expectation is with respect to the diffusion process given $(V_{t_0}, \ldots, V_{t_n}) = \mathbf{v}$. How efficiently would the methods in the paper apply when the likelihood is approximated and maximized using an MCMC missing data approach?

In passing it may be worth noticing that if the diffusion is a CIR model, Srinivasan (1988) and Clifford and Wei (1993) have established the equivalence between the Cox process and the process of death times of a simple immigration, birth and death process, which is easy to simulate. Incidentally, this model is a special case of the permanent process introduced in McCullagh and Møller (2005).

References

[1] Clifford, P. and Wei, G. (1993). The equivalence of the Cox process with squared radial Ornstein-Uhlenbeck intensity and the death process in a simple population model. *Annals of Applied Probability*, **3**, 863–873.

[2] McCullagh, P. and Møller, J. (2005). The permanent process. *Research Report R-2005-29*, Department of Mathematical Sciences, Aalborg University.

[3] Srinivasan, S. K. (1988). *Point Process Models of Cavity Radiation and Detection.* Griffin, London.

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