Discussion contribution: Lindgren, F., Rue, H. and Lindström, J. (2011). An explicit link between Gaussian fields and Gaussian Markov random fields: The stochastic partial differential equation approach. To appear in *J. R. Statist. Soc. B*, **73**.

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This important and impressing paper by Lindgren, Rue and Lindström (LRL) provides a computationally feasible approach for large spatial datasets analyzed by a hierarchical Bayesian model. It involves a latent Gaussian field, a parameter θ of low dimension, GMRF approximations to SPDEs for covariance functions, and the INLA-package for the computations instead of time consuming MCMC methods. Other recent papers by the authors (Simpson *et al.*, 2010; Bolin and Lindgren, 2011) compare the approach in LRL with kernel convolution methods (process convolution approaches) and covariance tapering methods, and conclude that the GMRF approximation to SPDEs is superior.

The Matérn covariance function plays a key-role, where e.g. in the planar case LRL assume that the shape parameter ν is a non-negative integer when considering the SPDE. Is their some link to the fact that this stationary planar covariance function is proportional to the mixture density of a zero-mean radially symmetric bivariate normal distribution $N_2(0, WI_2)$ with the variance W following an $\nu + 1$ times convolution of an exponential distribution?

Despite its popularity and flexibility for modelling different degrees of smoothness, is this 3-parameter Matérn covariance function really flexible enough for modelling large spatial datasets? Would a flexible non-parametric Bayesian approach be more appropriate for 'huge' spatial datasets, although this of course may be computationally slow? The dimension of θ may then be expected to be so high that INLA (Rue *et al.*, 2009) would not work; as the dimension of θ may even be varying, a reversible jump MCMC method (Green, 1995) may be needed when updating θ from its full conditional. When updating the Gaussian field from its full conditional (corresponding to a finite set of locations), a Metropolis-Hastings algorithm may apply (Roberts and Tweedie, 1996; Møller and Waagepetersen, 2004).

LRL do not discuss model checking. INLA provides quick estimates of the marginal posterior distributions of the Gaussian field and of θ. For model checking based on the joint posterior distribution, e.g. when comparing the data with simulations from the posterior predictive distribution, I presume MCMC algorithms still are needed.

Finally, using a triangulation for a finite element representation of a Guassian field is an appealing idea. For a spatial point pattern modelled by a log Gaussian Cox process (Møller *et al.*, 1998), I expect a regular triangulation should be used, since both the point pattern and the 'empty space' provide important information.

References

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