Exam for Algebra 1 at Aalborg University
For 3. semester Mat
Thursday, January 19th 2012, 8:30–13:30*.

You are allowed to use books and your notes. However calculators, computers, pdas or phones are not allowed.
A reasoned explanation should follow the solution of the exercises. Moreover, the intermediate steps leading to the solution should also be written down. You can write the exam in Danish or English.
The percentage following each exercise number stands for the exercise's value in the final mark.

Exercise 1 *(10%)* Find all solutions $x \in \mathbb{Z}$ to $93x \equiv 11 \pmod{64}$.

Exercise 2 *(20%)*

1. Let $\sigma \in S_{13}$,
$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 3 & 4 & 5 & 6 & 7 & 8 & 1 & 10 & 11 & 2 & 13 & 12 & 9 \end{pmatrix}.$$
Compute the order and the sign of σ.

2. Let $\sigma_1 = (1\ 4\ 2\ 5\ 3)$, $\sigma_2 = (1\ 3)(4\ 5) \in S_5$. Find $\tau \in S_5$ such that $\tau\sigma_1 = \sigma_2$.

3. Let $\sigma' = (1\ 5)(2\ 4) \in S_5$. Compute the number of inversions of σ' and write σ' as a product of the minimal number of simple transpositions.

Exercise 3 *(20%)* Let G be the quaternion group and $H = \{\mathbf{1}, -\mathbf{1}\}$. One has that $G = \{\mathbf{1}, -\mathbf{1}, \mathbf{i}, -\mathbf{i}, \mathbf{j}, -\mathbf{j}, \mathbf{k}, -\mathbf{k}\} \subset GL_2(\mathbb{C})$, where
$$\mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ \mathbf{i} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix},\ \mathbf{j} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ \mathbf{k} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

1. Prove that H is a subgroup in G.

2. Prove that H is normal in G.

3. Write the composition table for G/H.

Exercise 4 *(10%)* Let $G = \langle g \rangle$ with $\operatorname{ord}(g) = 24$ and $H = \langle g^6 \rangle$. Compute the order of the elements g^2H, g^3H, g^4H and g^5H in G/H.

Exercise 5 *(20%)* Let $G = D_3 = \{e, a, a^2, b, ba, ba^2\}$, where $\operatorname{ord}(a) = 3$, $\operatorname{ord}(b) = 2$ and $aba = b$. Write G as a disjoint union of conjugacy classes. Compute $Z(G)$.

Exercise 6 *(20%)* Let $s = \operatorname{lcm}(m, n)$. Show that $\mathbb{Z}/s\mathbb{Z}$ is isomorphic to a subgroup of $\mathbb{Z}/m\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z}$.

Husk at skrive jeres fulde navn på hver side af besvarelsen. Nummerer siderne, og skriv antallet af afleverede ark på 1. side af besvarelsen. God arbejdslyst.