$\mathcal{B}$: basis for $\mathbb{R}^n$

$$\vec{v} = B\,[\vec{v}]_{\mathcal{B}}$$

$$[\vec{v}]_{\mathcal{B}} = B^{-1}\,\vec{v}$$

<u>EKS</u>

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

Er $\mathcal{B}$ en basis for $\mathbb{R}^3$?

Hvis Ja: omregn mellom $\vec{v}$ og $[\vec{v}]_{\mathcal{B}}$

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} B & I_3 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\text{rref}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right]$$

$\mathcal{B}$ er basis da $\text{rref}(B) = I_3$ og

$$B^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

Lad $\vec{v} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$

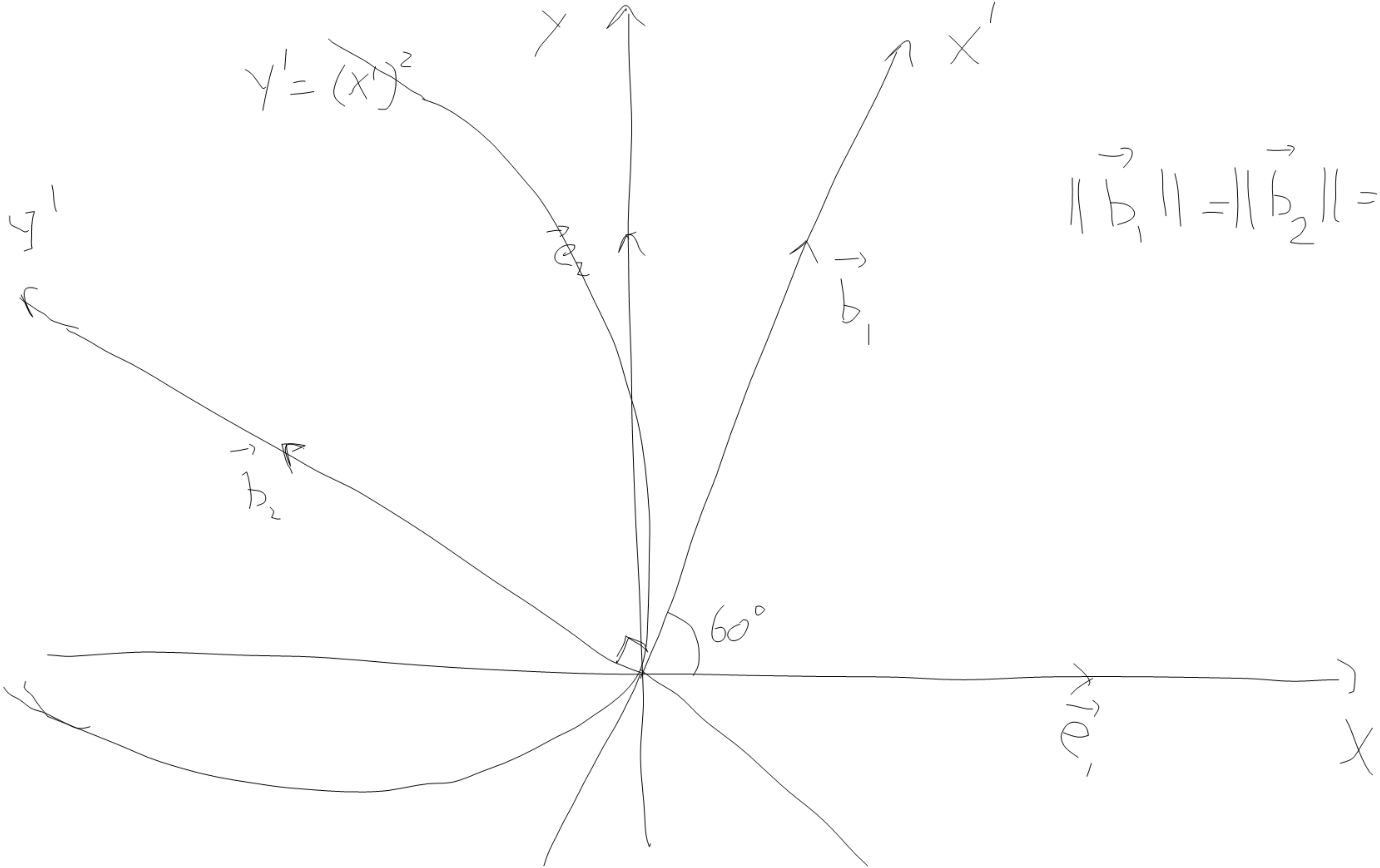
Så er $[\vec{v}]_B = B^{-1}\vec{v} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 5 \end{bmatrix}$

Lad $\vec{u}$ være vektoren med $[\vec{u}]_B = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$

Så är $\vec{u} = B\,[\vec{u}]_{B} =$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 5 \end{bmatrix}$$

EKS i $\mathbb{R}^2$



$$\|\vec{b}_1\| = \|\vec{b}_2\| = 1$$

$$B = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 \end{bmatrix} = A_{60^\circ} = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$\vec{b}_1 = A_{60^\circ} \vec{e}_1 = \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\vec{b}_2 = A_{60^\circ} \vec{e}_2 = \begin{bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$\mathcal{B} = \{ \vec{b}_1 , \vec{b}_2 \}$$

Lad $\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$

Koordinatvektor:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = [\vec{v}]_{\mathcal{B}} = B^{-1} \vec{v}$$

$$B^{-1} = A_{60^\circ}^{-1} = A_{-60^\circ} = \begin{bmatrix} \cos -60^\circ & -\sin -60^\circ \\ \sin -60^\circ & \cos -60^\circ \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = B^{-1}\vec{v} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{2}x + \frac{\sqrt{3}}{2}y \\ -\frac{\sqrt{3}}{2}x + \frac{1}{2}y \end{bmatrix}$$

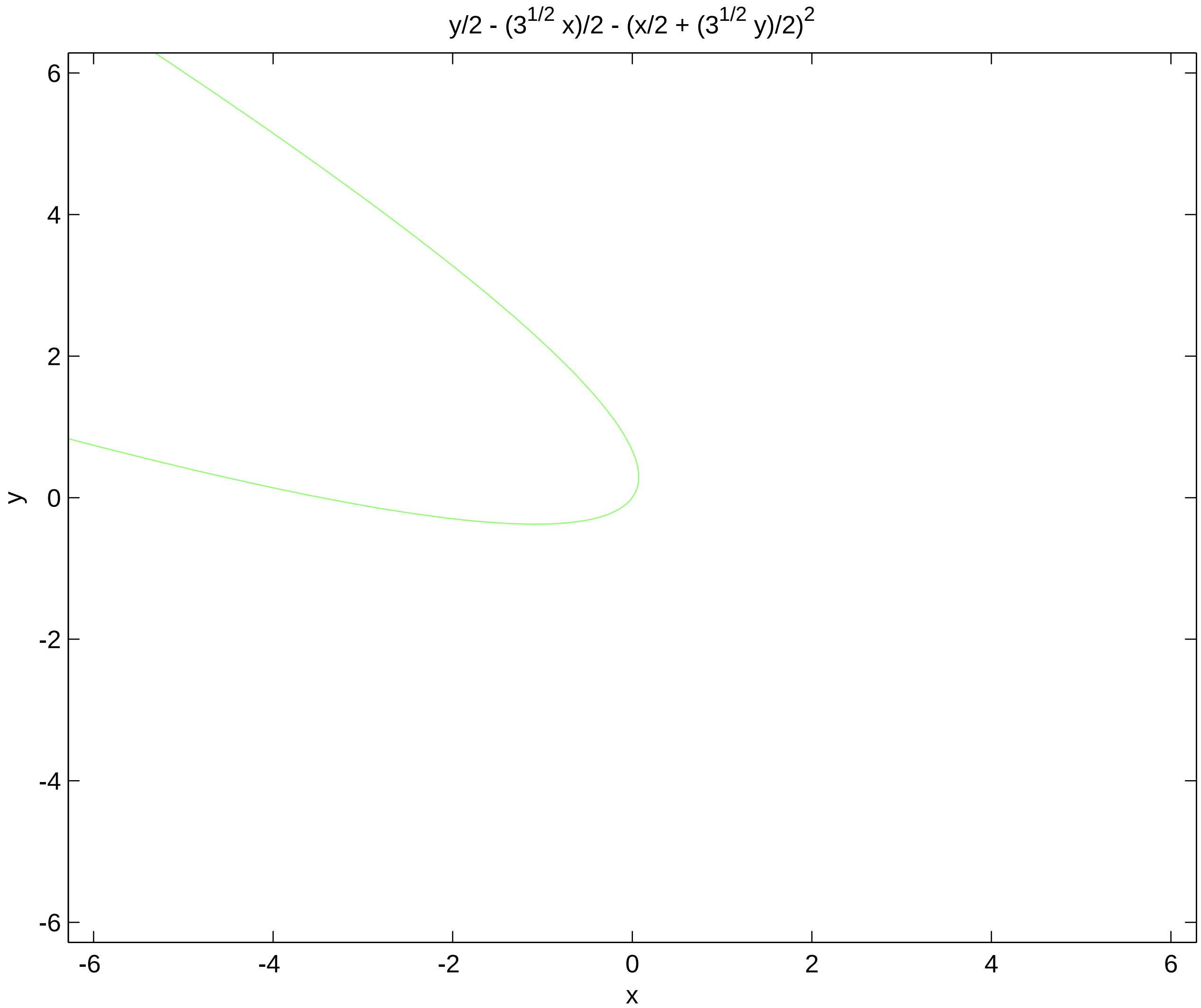
$(\ast)$ $x' = \frac{1}{2}x + \frac{\sqrt{3}}{2}y$ og $y' = -\frac{\sqrt{3}}{2}x + \frac{1}{2}y$

Parablens ligning $\quad y' = (x')^2$

Indsæt $(*)$:

$$-\frac{\sqrt{3}}{2}x + \frac{1}{2}y = \left(\frac{1}{2}x + \frac{\sqrt{3}}{2}y\right)^2$$

```
>> syms  x y
>>
>> xx = 1/2 * x  +  sqrt(3)/2 * y

xx =

x/2 + (3^(1/2)*y)/2

>> yy = -sqrt(3)/2 * x  +  1/2 * y

yy =

y/2 - (3^(1/2)*x)/2

>> parabel = yy-xx^2

parabel =

y/2 - (3^(1/2)*x)/2 - (x/2 + (3^(1/2)*y)/2)^2

>> ezplot(parabel)
>>
```



y/2 - (3^{1/2} x)/2 - (x/2 + (3^{1/2} y)/2)^2
6
4
2
0
-2
-4
-6
y
-6
-4
-2
0
2
4
6
x