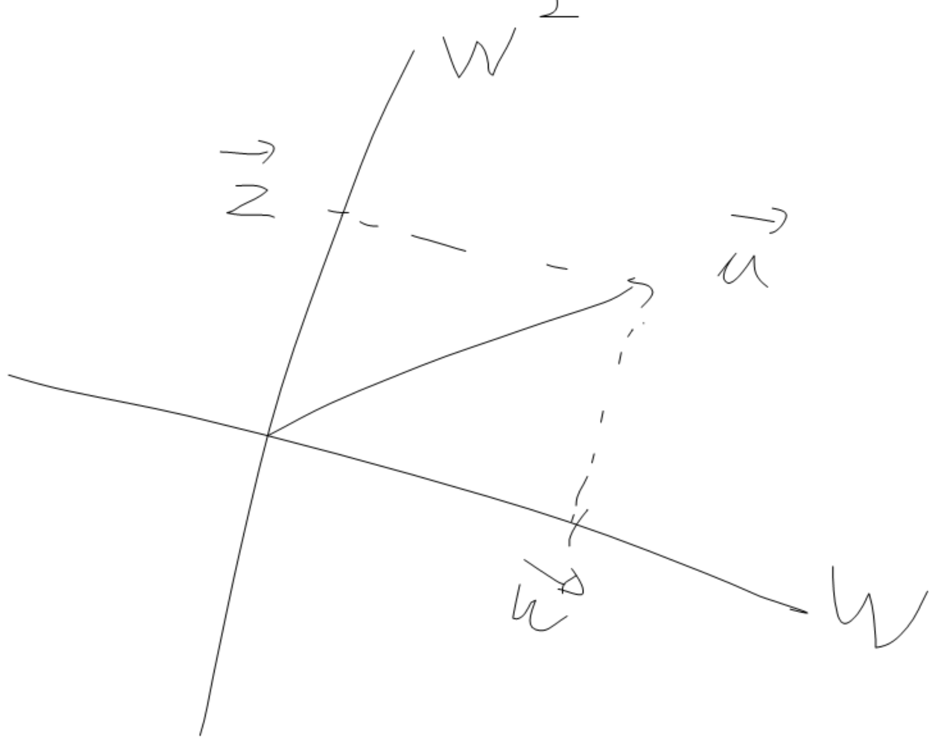
W : underrom af R^n

Ortogonal projektion af $\vec{u}$ på W

$$\vec{w} = U_W(\vec{u})$$



Hvis $\{\vec{v}_1, \dots, \vec{v}_k\}$ er ortonormal basis for W

og $\{\vec{v}_{k+1}, \dots \vec{v}_n\}$ er ortonormal basis for $W^\perp$

Så er $\{\vec{v}_1 \dots \vec{v}_k \dots \vec{v}_n\}$ er ortonormal basis for $\mathbb{R}^n$

og $\vec{u} = \underbrace{(\vec{u}\cdot\vec{v}_1)\vec{v}_1 + \dots + (\vec{u}\cdot\vec{v}_k)\vec{v}_k}_{\vec{w} \text{ i } W} +$

$\underbrace{(\vec{u}\cdot\vec{v}_{k+1})\vec{v}_{k+1} + \dots + (\vec{u}\cdot\vec{v}_n)\vec{v}_n}_{\vec{z} \text{ i } W^\perp}.$

Altså

$U_W(\vec{u}) = (\vec{u}\cdot\vec{v}_1)\vec{v}_1 + \dots + (\vec{u}\cdot\vec{v}_k)\vec{v}_k$

EKS

$$W = \text{span}\left\{ \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}, \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix} \right\} = \{\vec{v}_1, \vec{v}_2\}$$

$$\vec{u} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

Find $U_W(\vec{u}) = (\vec{u} \cdot \vec{v}_1)\vec{v}_1 + (\vec{u} \cdot \vec{v}_2)\vec{v}_2 =$

$$\left(\frac{2}{3} + \frac{2}{3} + \frac{2}{3}\right)\vec{v}_1 + \left(\frac{4}{3} + \frac{1}{3} - \frac{2}{3}\right)\vec{v}_2 =$$

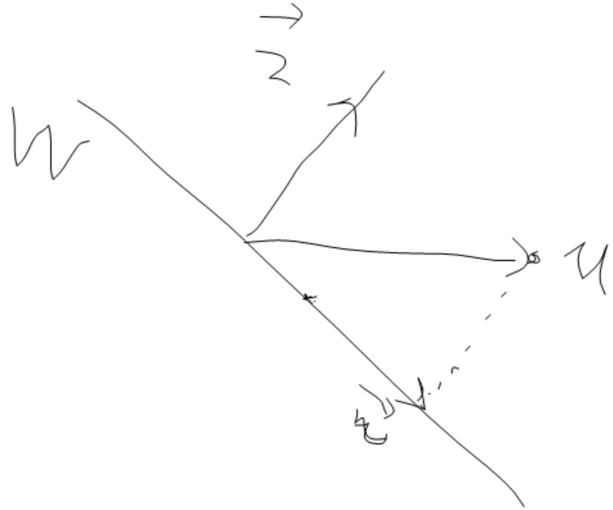
$$2\vec{v}_1 + 1 \cdot \vec{v}_2 = 2\begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix} + \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{4}{3} \\ \frac{5}{3} \\ \frac{2}{3} \end{bmatrix}$$

$$\vec{z} = U_{W^\perp}(\vec{u}) = \vec{u} - \vec{w} = \vec{u} - U_W(\vec{u}) =$$

$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} \frac{4}{3} \\ \frac{5}{3} \\ \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$\|\vec{z}\| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = 1$$

= afstand fra $\vec{u}$ til W.



$U_W(\vec{u})$ er den vektor i W der er nærmest $\vec{u}$.

$U_W : \mathbb{R}^n \to \mathbb{R}^n$ er en

linear transformation (linear operator)

Standard matrix for U_W: P_W

$\{\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_k\}$ basis for W

(ikke ortogonal)

$C = [\vec{b}_1 \ \vec{b}_2 \ \ldots \ \vec{b}_k]$: $n \times k$ matrix

Så ur $P_W = \underbrace{\underset{n\times k}{C}\,\underset{k\times k}{(C^TC)^{-1}}\,\underset{k\times n}{C^T}}_{n\times n}$

<u>EKS</u> i $\mathbb{R}^4$

$$W = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

$$C = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$C^T C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$(C^T C)^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$P_W = C\,(C^T C)^{-1} C^T = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} C^T$$

$$\begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$U_W(\vec{u}) = P_W \vec{u} = \begin{bmatrix} \frac{3}{2} \\ \frac{3}{2} \\ \frac{7}{2} \\ \frac{7}{2} \end{bmatrix} = \frac{3}{2}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \frac{7}{2}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$