4.4

V: et underrum af $\mathbb{R}^n$

$\mathcal{B} = \{\vec{b}_1, \ldots, \vec{b}_p\}$ basis for V.

Dvs: span $\mathcal{B} = V$, altså

for enhver $\vec{v} \in V$ findes $c_1 \ldots c_p$

så $\vec{v} = c_1\vec{b}_1 + \ldots + c_p\vec{b}_p$

Hvis $\vec{v} = a_1\vec{b}_1 + \ldots + a_p\vec{b}_p$

så er (1. ligning − 2. ligning):

$$\vec{0} = (c_1 - a_1)\vec{b}_1 + \dots + (c_p - a_p)\vec{b}_p$$

Da $\vec{b}_1, \dots, \vec{b}_p$ er lineært uafhængige

så $\quad c_1 - a_1 = 0 \quad \dots \quad c_p - a_p = 0$

$$a_1 = c_1 \qquad\qquad a_p = c_p$$

$\vec{v}$ kan altså entydigt skrives som linear kombination af basis-vektorer.

Hvis $V = \mathbb{R}^n$ så er $p = n$ og

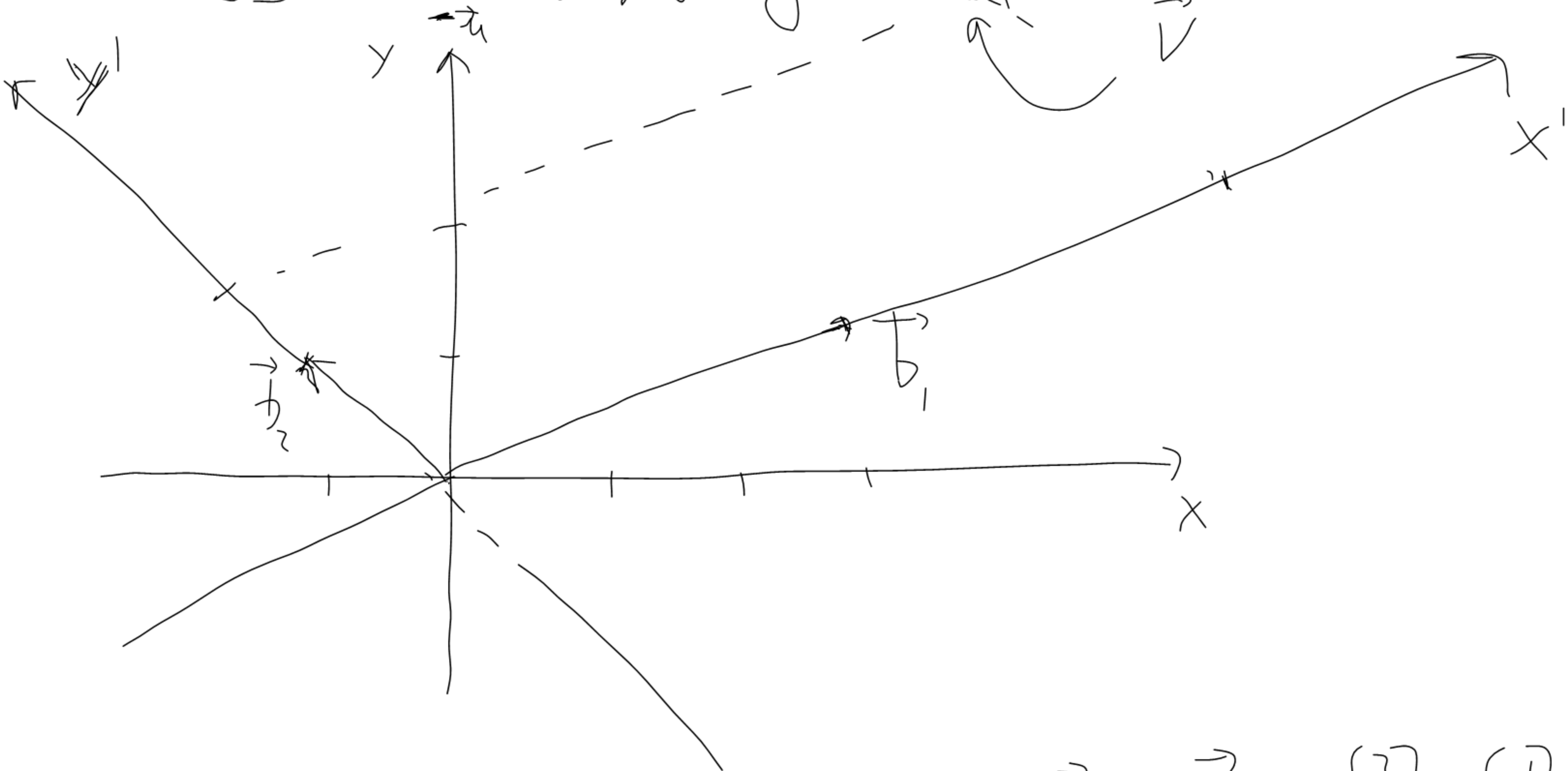
vektoren $\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$ kalles koordinatvektor for $\vec{v}$ m.h.t. $\mathcal{B}$

Skrives $\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = [\vec{v}]_{\mathcal{B}}$

<u>Eks</u> $V = \mathbb{R}^2$

$$\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\} = \{ \vec{b}_1, \vec{b}_2 \}$$

$\mathcal{B}$ er basis for $\mathbb{R}^2$



Hvis $[\vec{v}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ så $\vec{v} = 2\vec{b}_1 + 2\vec{b}_2 = 2\begin{bmatrix} 3 \\ 1 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

Hvis $\vec{u} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$ Find $[\vec{u}]_{\mathcal{B}}$

Løs $x' \begin{bmatrix} 3 \\ 1 \end{bmatrix} + y' \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$

$$\begin{bmatrix} 3 & -1 & 0 \\ 1 & 1 & 4 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 1 & 4 \\ 3 & -1 & 0 \end{bmatrix} \xrightarrow{R_2 - 3R_1 \to R_2}$$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & -4 & -12 \end{bmatrix} \xrightarrow[R_1 - R_2 \to R_1]{-\frac{1}{4}R_2 \to R_2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

Dvs
$x' = 1$
$y' = 3$

$$[\vec{u}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

EKS Standardbasen for $\mathbb{R}^n$

$$\mathcal{E} = \{\vec{e}_1, \vec{e}_2, \dots, \vec{e}_n\}$$

Hvis $\vec{v} \in \mathbb{R}^n$ så er $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

$$[\vec{v}]_{\mathcal{E}} = \vec{v}$$

da $\vec{v} = v_1\vec{e}_1 + v_2\vec{e}_2 + \dots + v_n\vec{e}_n$

Teori

$$\mathcal{B} = \{ \vec{b}_1 \ \dots \ \vec{b}_n \} \quad \text{for } \mathbb{R}^n$$

$$B = \begin{bmatrix} \vec{b}_1 & \dots & \vec{b}_n \end{bmatrix} \qquad n \times n \text{ matrix}$$

B har pivot i alle rækker / søjler

da $\mathcal{B}$ er basis,

Altså B er invertibel.

Hvis $[\vec{v}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$ så er

$$\vec{v} = c_1\vec{b}_1 + c_2\vec{b}_2 + \dots + c_n\vec{b}_n =$$

$$\begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \dots & \vec{b}_n \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = B\,[\vec{v}]_{\mathcal{B}}$$

Altså (setning 4.11):

$$\vec{v} = B\,[\vec{v}]_{\mathcal{B}}$$

$$B^{-1}\vec{v} = [\vec{v}]_{\mathcal{B}}$$

EKS

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \qquad B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Er $\mathcal{B}$ basis for $\mathbb{R}^3$?

Hvis ja: omregning mellom $\vec{v}$ og $[\vec{v}]_{\mathcal{B}}$?

$$[B \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\text{rref}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right]$$

B har pivot i alle søjler. $\mathcal{B}$ er lineært uafh.

$\mathcal{B}$ er altså basis for $\mathbb{R}^3$

$$B^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

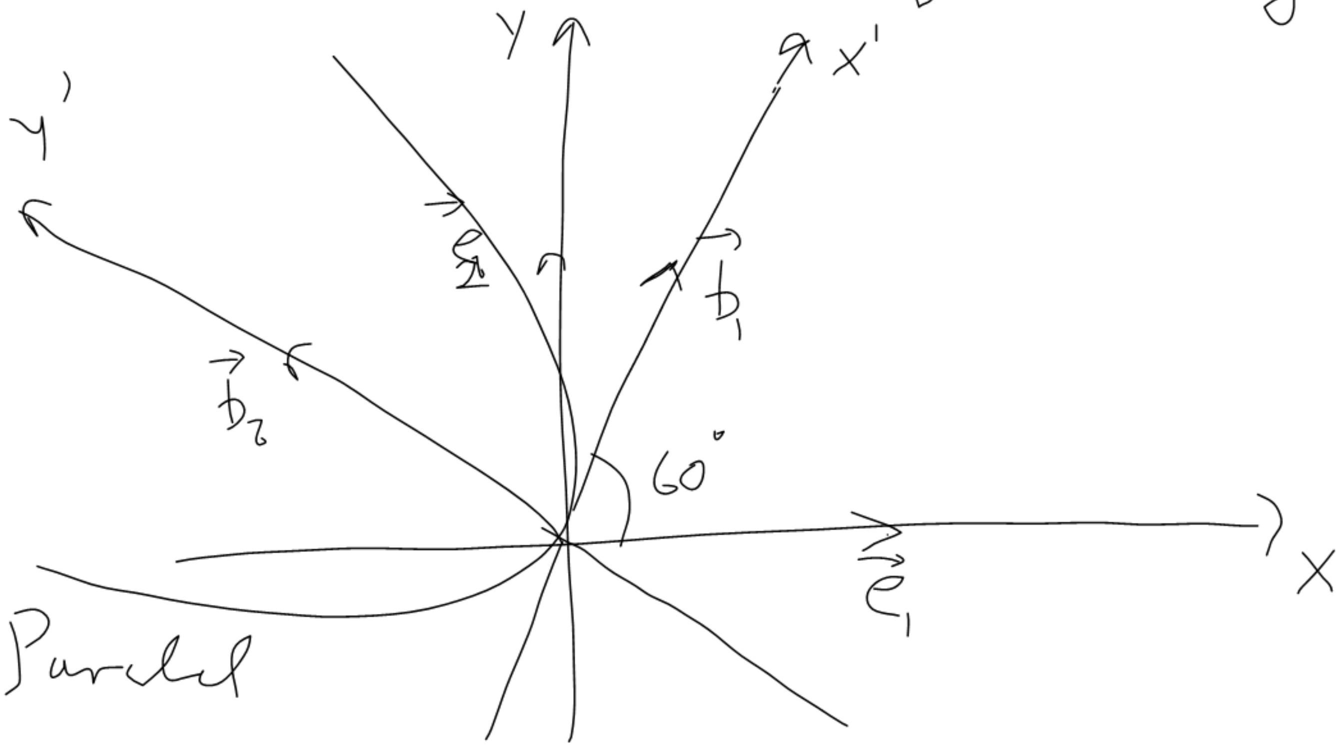
Koordinatvektor for $\vec{v}$ m.h.t. $\mathcal{B}$

$$[\vec{v}]_{\mathcal{B}} = B^{-1}\vec{v} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

Hvis $[\vec{u}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ så er

$$\vec{u} = B[\vec{u}]_{\mathcal{B}} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$$

EKS Koordinatsystem og kurve i $\mathbb{R}^2$



Rotationsmatrix

$$A_{60^\circ} = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} \vec{b}_1 & \vec{b}_2 \end{bmatrix} = B \quad , \quad \mathcal{B} = \{ \vec{b}_1 , \vec{b}_2 \}$$

$$B^{-1} = A_{-60^\circ} = \begin{bmatrix} \cos -60^\circ & -\sin -60^\circ \\ \sin -60^\circ & \cos -60^\circ \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} \vec{e}_1 \end{bmatrix}_{\mathcal{B}} = B^{-1} \vec{e}_1 = \begin{bmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix}$$

Parabel med ligning i $x'y'$ koordinatsyst.

$$y' = x'^2$$

Find ligning i xy-koordinatsystem

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}_{\mathcal{B}} = B^{-1}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$x' = \frac{1}{2}x + \frac{\sqrt{3}}{2}y$$

$$y' = -\frac{\sqrt{3}}{2}x + \frac{1}{2}y$$

$$y' = x'^2$$

$$\Updownarrow$$

$$-\frac{\sqrt{3}}{2}x + \frac{1}{2}y = \left(\frac{1}{2}x + \frac{\sqrt{3}}{2}y\right)^2$$