

PhD Course  
Fourier Analysis  
Problem Set 1.

This is the first problem set for the course, to be handed in no later than March 11, 2005.

We consider the following function:

$$f(t) = \begin{cases} \sin(2t) & \text{for } t \in [-\pi, 0], \\ \cos(\frac{1}{2}t) & \text{for } t \in (0, \pi). \end{cases} \quad (1)$$

This function is extended to all of  $\mathbf{R}$  as a  $2\pi$ -periodic function.

1. Show that this function is piecewise smooth, as defined in BNB Section 4.3. Compute the values  $f'_{\pm}(t)$  at the three points  $t = -\pi$ ,  $t = 0$ , and  $t = +\pi$ .
2. Compute the Fourier series of the function  $f(t)$ .
3. Discuss the pointwise convergence properties of this Fourier series. At which points in  $[-\pi, \pi]$  does it converge to the function values? What does it converge to at  $t = 0$ .
4. (Optional) Illustrate your results and your discussion with various plots.

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