PhD Course
Fourier Analysis
Problem Set 2.

This is the second problem set for the course, to be handed in no later than March 11, 2005.

Consider the following two functions

$$f(t) = \begin{cases} t+\pi & \text{for } t \in [-\pi, 0], \\ \pi - t & \text{for } t \in (0, \pi), \end{cases} \tag{1}$$

$$g(t) = \begin{cases} t+\pi & \text{for } t \in [-\pi, 0], \\ t - \pi & \text{for } t \in (0, \pi). \end{cases} \tag{2}$$

The two functions are extended to all of $\mathbf{R}$ as 2π-periodic functions. Explain that the extension of f is continuous, while the extension of g has jump discontinuities at all points $t = 2\pi k$, $k \in \mathbf{Z}$.

The Fourier series are given by

$$f(t) \sim \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{4}{\pi(2n-1)^2} \cos((2n-1)t)$$

and

$$g(t) \sim \sum_{n=1}^{\infty} \frac{-2}{n} \sin(nt).$$

Using these results, discuss convergence rates using numerical experiments, either in Matlab, or in Maple.

How many terms are needed in the partial sum sf_N of f to get

$$|f(0) - sf_N(0)| < 0.01?$$

Will the Cesaro mean with the same number of terms give a better result?

For the function g we try to approximate the value $g(1/\sqrt{2})$. Thus find the number of terms needed to get

$$|g(1/\sqrt{2}) - sg_N(1/\sqrt{2})| < 0.01,$$

where sg_N denotes the partial sum for g. What about the Cesaro mean in this case?

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