Matematisk modellering og numeriske metoder

Hints to the exercises related to Lecture 8

Morten Grud Rasmussen

October 9, 2013

Section 11.1

Exercise 1

- Just use the definition.
- It can be helpful to sketch the functions.

Exercise 11

- The main idea here is to use partial integration.
- This is something you can only do for yourselves but it is nevertheless important.

Exercise 13

- Look at the sketch or graph from Exercise 9. It should be clear that the integralsfrom $-\pi$ to 0 and from 0 to π in the calculation of a_0 cancels, leaving $a_0 = 0$.
- Split the integral in the formulas for a_n and b_n in two; from $-\pi$ to 0 and from 0 to π.
- This is a give-away: an anti-derivative of $x\cos(nx)$ is $\frac{nx\sin(nx)+\cos(nx)}{n^2}$.
- The same for $x\sin(nx)$: $\frac{\sin(nx)-nx\cos(nx)}{n^2}$
- Remember what $\sin$ and $\cos$ evaluate to at $n\pi$!
- If you do it right, your b_n's should be $2, 0, \frac{2}{3}, 0, \frac{2}{5}$ etc. for $n = 1, 2, 3, 4, 5$ etc. and your a_n's should be $\frac{4}{\pi}, 0, \frac{4}{9\pi}, 0, \frac{4}{25\pi}$ etc. for $n = 1, 2, 3, 4, 5$ etc.

Exercise 17

- This exercise is much easier after Lecture 9!
- The function is given by

$$f(x) = \begin{cases} \pi + x & \text{for } -\pi < x < 0 \\ \pi - x & \text{for } 0 < x < \pi \end{cases}.$$

- The b_n's are all 0.
- All calculations are similar to calculations found in the hints for Exercise 13.

Exercise 21

- Exactly as above, this is much easier after Lecture 9.
- The function is given by

$$f(x) = \begin{cases} -\pi - x & \text{for } -\pi < x < 0 \\ \pi - x & \text{for } 0 < x < \pi \end{cases}$$

- This time, the a_n's are all 0.
- Again, all calculations have basically been done when solving Exercise 13.
- The b_n's are given by $b_n = \frac{2}{n}$.

Exercise 23

- Since all a_n's are 0, and all discontinuity points of the function are at x's where $\sin(nx)$ is 0, the series can be manually evaluated at the discontinuity points!
- Find the value at these points (actually, there is only one...).
- Find the left-hand and right-hand limits at the discontinuity point.