

Opgave 15 (Bevis for sætning 3.2)

Sætning 0.0.1 *Lad (V, N) være et normeret vektorrum. Så er (V, d_N) et metrisk rum.*

Bevis

Vi ved

$$\begin{aligned}N(x) &= 0 \Leftrightarrow x = 0, \\N(ax) &= |a|N(x), \\N(x + y) &\leq N(x) + N(y), \\d_N(x, y) &= N(x - y),\end{aligned}$$

for $x, y \in V$. Vi skal vise at d_N opfylder

1. $d_N(x, y) = 0 \Leftrightarrow x = y$,
2. $d_N(x, y) = d_N(y, x)$,
3. $d_N(x, y) \leq d(x, z) + d(z, y)$,

for $x, y, z \in V$.

Bevis for 1: Lad $d_N(x, y) = 0$. Så

$$N(x - y) = d_N(x, y) = 0 \Leftrightarrow x - y = 0 \Leftrightarrow x = y.$$

Bevis for 2:

$$d_N(x, y) = N(x - y) = |-1|N(y - x) = N(y - x) = d_N(y, x).$$

Bevis for 3:

$$d_N(x, y) = N(x - y) = N[x - z + (z - y)] \leq N(x - z) + N(z - y) = d(x, z) + d(z, y).$$