ORDINÆR EKSAMEN, MATEMATISK MODELLERING OG NUMERISKE METODER

Opgave 1. $y(x) = x$

Opgave 2. $y(x) = \frac{3}{4x} + \frac{x^3}{4}$

Opgave 3. $y(x) = 1 + cx^4$, $c \in \mathbb{R}$.

Opgave 4. $y(t) = c_1 e^{-2t} + c_2 e^{6t} + 2e^{-5t}$, $c_1, c_2 \in \mathbb{R}$.

Opgave 5. $y(t) = c_1 \sin(t) + c_2 + e^{\sin(t)}$, $c_1, c_2 \in \mathbb{R}$.

Opgave 6. $s^2 Y(s) - 12Y(s) + 4(sY(s) - 10) - 10s - 3 = 66\frac{s+5}{(s+5)^2+4}$

Opgave 7. $y(t) = c_1 \binom{1}{1} e^{5t} + c_2 \binom{1}{2} e^{4t}$

Opgave 8. $\frac{3}{4}\sin(x) - \frac{1}{4}\sin(3x)$.

Opgave 9. $\sum_{n=1}^{\infty} \frac{2(-1)^{n+1} - (1-(-1)^n)}{n} \sin(nx) = -\sum_{n=1}^{\infty} \frac{\sin(2nx)}{n}$

Opgave 10. $u(x,t) = \sum_{n=1}^{9} \frac{4}{2n-1} \sin(\frac{(2n-1)\pi}{3} x) e^{-\frac{4^2(2n-1)^2\pi^2}{3^2} t}$

Opgave 11. Yes, the derivative is clearly strictly less than 1 in absolute value around $\cos(x) = x$

Opgave 12. $p_5(x) = (x-2)(x-3)(x-5)(x-7)(x-13)$

Opgave 13. $J_2^S = 280$, $\varepsilon_2^S \approx \frac{1}{15}(J_2^S - J_1^S) = \frac{8}{15}$, $\varepsilon_2^S = -\frac{4-0}{2880} 2^2(-24) = \frac{8}{15}$ (in fact, the latter is not an estimate but a precise result, since the fourth derivative is a constant, hence the equality sign).

Opgave 14. $y_{n+1} = \frac{y_n}{1+10hx_{n+1}} = \frac{y_n}{1+10h(x_n+h)} = \frac{y_n}{1+5(x_n+0.5)}$

Opgave 15. -1.08713

Opgave 16. The solution is represented by a *function* which is a *vector in an abstract vector space*.