

**Opgave 1** (25 POINT)

- (a)  $y_h(x) = c_1 e^x + c_2 x e^x$   
 (b)  $y_p(x) = x^2 e^x + x^3 e^x$   
 (c)  $y_g = y_h + y_p$   
 (d)  $y_0(x) = e^x + x e^x + x^2 e^x + x^3 e^x$   
 (e)  
 (f)  $f(x) = y_0(x) e^{-x}$   
 (g)

$$a_0(h) = 1 + \frac{\pi^2}{3}$$

$$a_n(h) = (-1)^n \frac{4}{n^2} \quad \text{for } n \neq 2$$

$$b_n(h) = (-1)^n \frac{12 - 2n^2(1 + \pi^2)}{n^3}.$$

$$a_2(h) = 0.$$

**Opgave 2** (25 POINT)

- (a)  $u_t = c^2 u_{xx}$   
 (b)  $u(x, 0) = f(x)$   
 (c)  $u_x(0, t) = u_x(1, t) = 0$ .  
 (d)  $\lambda_n = cn\pi$ ,  $u_n(x, t) = a_n \cos(n\pi x) e^{-c^2 n^2 \pi^2 t}$ ,  $n \in \mathbb{N}_0$   
 (e)  $u(x, t) = \sum_{n=0}^{\infty} u_n(x, t)$   
 (f)  $a_0 = \int_0^1 f(x) dx$ ,  $a_n = 2 \int_0^1 f(x) \cos(n\pi x) dx$   
 (g)  $u(x, t) = u_0$ .  
 (h)  $\lim_{t \rightarrow \infty} u(x, t) = u_1$

**Opgave 3** (10 POINT)

- (a)  $y_1 = -1.7889929$ ,  $k_1 = 0.270151153$ ,  $k_2 = 0.220122672$ ,  $k_3 = 0.208825117$  og  $k_4 = 0.137995908$   
 (b)  $\frac{1}{15}(-1.6715985 - (-1.6660306))$   
 (c)  $y_5 = -2.8422375$   
 (d)  $\frac{1}{15} = (-2.8422375 - (-2.7581849))$