# Opgave 1 (25 POINT)

(a) $y_h(x) = c_1 e^x + c_2 x e^x$

(b) $y_p(x) = x^2 e^x + x^3 e^x$

(c) $y_g = y_h + y_p$

(d) $y_0(x) = e^x + x e^x + x^2 e^x + x^3 e^x$

(e)

(f) $f(x) = y_0(x) e^{-x}$

(g)

$$
\begin{aligned}
a_0(h) &= 1 + \frac{\pi^2}{3} \\
a_n(h) &= (-1)^n \frac{4}{n^2} \quad \text{for} \quad n \neq 2 \\
b_n(h) &= (-1)^n \frac{12 - 2n^2(1+\pi^2)}{n^3}.
\end{aligned}
$$

$a_2(h) = 0.$

# Opgave 2 (25 POINT)

(a) $u_t = c^2 u_{xx}$

(b) $u(x,0) = f(x)$

(c) $u_x(0,t) = u_x(1,t) = 0.$

(d) $\lambda_n = cn\pi$, $u_n(x,t) = a_n \cos(n\pi x) e^{-c^2 n^2 \pi^2 t}$, $n \in \mathbb{N}_0$

(e) $u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$

(f) $a_0 = \int_0^1 f(x)\,\mathrm{d}x$, $a_n = 2\int_0^1 f(x)\cos(n\pi x)\,\mathrm{d}x$

(g) $u(x,t) = u_0.$

(h) $\lim_{t\to\infty} u(x,t) = u_1$

# Opgave 3 (25 POINT)

(a) $y_1 = -1.7889929$, $k_1 = 0.270151153$, $k_2 = 0.220122672$, $k_3 = 0.208825117$ og $k_4 = 0.137995908$

(b) $-1.67658$

(c) $y_{10} = -1.6715985$, $k_1 = 0.376789944$, $k_2 = 0.396310600$, $k_3 = 0.393316264$ og $k_4 = 0.409645913$

(d) $\frac{1}{15}(-1.6715985 - (-1.6660306))$

(e) $y_5 = -2.8422375$

(f) $\frac{1}{15} = (-2.8422375 - (-2.7581849))$

(g) $\tilde{y}_6 = -2.6751898$

(h) $\tilde{y}_{10} = -1.666196$ og $y_{10} = -1.6719068$

# Opgave 4 (25 POINT)

(a) $\mathcal{L}(f)(s) = \frac{\Gamma(e-1)}{s^{e-1}}$

(b) $\mathcal{L}(gf)(s) = \frac{\Gamma(e-1)}{(s-1)^{e-1}}$

(c) $\mathcal{L}(h)(s) = (e-1)e\frac{\Gamma(e-1)}{(s-1)^{e-1}}$

(d) $\mathcal{L}(y'')(s) = s^2Y(s)$, $\mathcal{L}(Y')(s) = sY(s)$

(e) $s^2Y(s) = 2sY(s) - Y(s) + (e-1)e\frac{\Gamma(e-1)}{(s-1)^{e-1}}$

(f) $Y(s) = \frac{\Gamma(e+1)}{(s-1)^{e+1}}$

(g) $t^{a-1}$

(h) $e^t t^e$