Opgave 1 (25 POINT)

(a) $y_h(x) = c_1 \sin(7x) + c_2 \cos(7x)$

(b) $y_p(x) = \frac{3}{2}\cos(2x) + \frac{1}{2}$

(c) $y_g = y_h + y_p$

(d) $y_0(x) = \sin(7x) + \frac{3}{2}\cos(2x) + \frac{1}{2}$

(e)

(f) $b_4(f) = a_7(f) = 0$, $a_2(f) = \frac{3}{2}$.

(g) $x \in \mathbb{R}$

Opgave 2 (25 POINT)

(a) $u(x,t) = a\sin(\pi x)e^{-\pi^2 t} + b\sin(3\pi x)e^{-9\pi^2 t}$

(b) $u_t(x,0) = -9\pi^2(\sin(\pi x) + \sin(3\pi x))$

(c) Temperaturen falder overalt til tiden $t = 0$

(d) $x_1 = \frac{1}{6}$ og $x_2 = \frac{1}{2}$

(e) Temeraturen stiger i x_1 til tiden $t = 0$

(f) $2 = e^{8\pi^2 t} \Leftrightarrow \log(2) = 8\pi^2 t \Leftrightarrow t = \frac{\log(2)}{8\pi^2}$

(g) Temperaturen ændres ikke i $\frac{5}{6}$ til tiden $t = t_0$.

Opgave 3 (25 POINT)

(a) $y_1 = 0.71544821$

(b) 6.66694599

(c) $y_{10} = 6.66941032$

(d) $\frac{1}{15}(6.66921218 - 6.66941032)$

(e) $y_5 = 2.88048846$

(f) $\frac{1}{15}(2.88048846 - 2.92147696) = -0.002732566$

(g) $\tilde{y}_6 = 3.62511069$

(h) $\tilde{y}_{10} = 6.66671799$ og $y_{10} = 6.66641926$

Opgave 4 (25 POINT)

(a) $x(\log(y(x)) - \frac{1}{2}\log(x^2+1) - 1) + \frac{x^2+1}{y(x)}y'(x) = 0$

(b) $\frac{\partial M_1}{\partial y}(x,y) = \frac{x}{y} \neq \frac{\partial N_1}{\partial x}(x,y) = \frac{2x}{y}$

(c) $\frac{x}{x^2+1}(\log(y(x) - \frac{1}{2}\log(x^2+1) - 1) + \frac{1}{y(x)}y'(x) = 0$

(d) $\frac{\partial M_2}{\partial y}(x,y) = \frac{x}{y(x^2+1)} \neq \frac{\partial N_2}{\partial x}(x,y) = 0$

(e) $R(x,y) = \frac{1}{N_2(x,y)}(\frac{\partial M_2}{\partial y}(x,y) - \frac{\partial N_2}{\partial x}) = \frac{x}{x^2+1}$ – afhænger ikke af y

(f) $F(x) = \exp\int^x R(x_1,y)\,\mathrm{d}x_1 = \exp(\frac{1}{2}\log(x^2+1)) = \sqrt{x^2+1}$

(g) $u(x,y) = \sqrt{x^2+1}(\log(y) - \frac{1}{2}\log(x^2+1))$

(h) $c = 1$, $y(x) = e^{\frac{1}{\sqrt{x^2+1}}}\sqrt{x^2+1}$