Exam 2011

Mathematics for Multimedia Applikations
AAU-Cph, Medialogy

1. June 2011

Formalities

This exam set consists of 8 pages. There are 10 problems containing 34 sub-problems in total. Books and notes are allowed but *no electronic devices* such as calculators, computers or cell phones are permitted.

A number of points is indicated for every sub-problem. The sum of these points equals 100.

Date and time for the exam: 1. June, 9:00 - 13:00

You must indicate the following on each page:

- Full name
- Study number
- Page number

On the first page, you must indicate

- The total number of pages.

Remark that special values for sine and cosine are added as an appendix.

Good luck!

Problems

Problem 1.

1.a. (3 points) Find the derivative of the function $x^3 + 2x\sin(x)$ with respect to x.

1.b. (3 points) Let $f(x) = 4\ln(x^3 + 2x)$. Calculate $f'(x)$.

1.a. $3x^2 + 2\sin(x) + 2x\cos(x)$ 1.b. $f'(x) = \dfrac{12x^2+8}{x^3+2x}$

Problem 2. Let $f(x) = x + 2\cos(x)$. The graph of this function appears in figure 1.

2.a. (2 points) Compute $f'(x)$. $f'(x) = 1 - 2\sin(x)$

2.b. (3 points) Find an x such that $f'(x) = 0$. $x = \frac{\pi}{6}$

2.c. (5 points) Describe all x such that $f'(x) = 0$. $x = \frac{\pi}{6} + 2p\pi \vee x = \frac{5\pi}{6} + 2p\pi, p \in \mathbb{Z}$

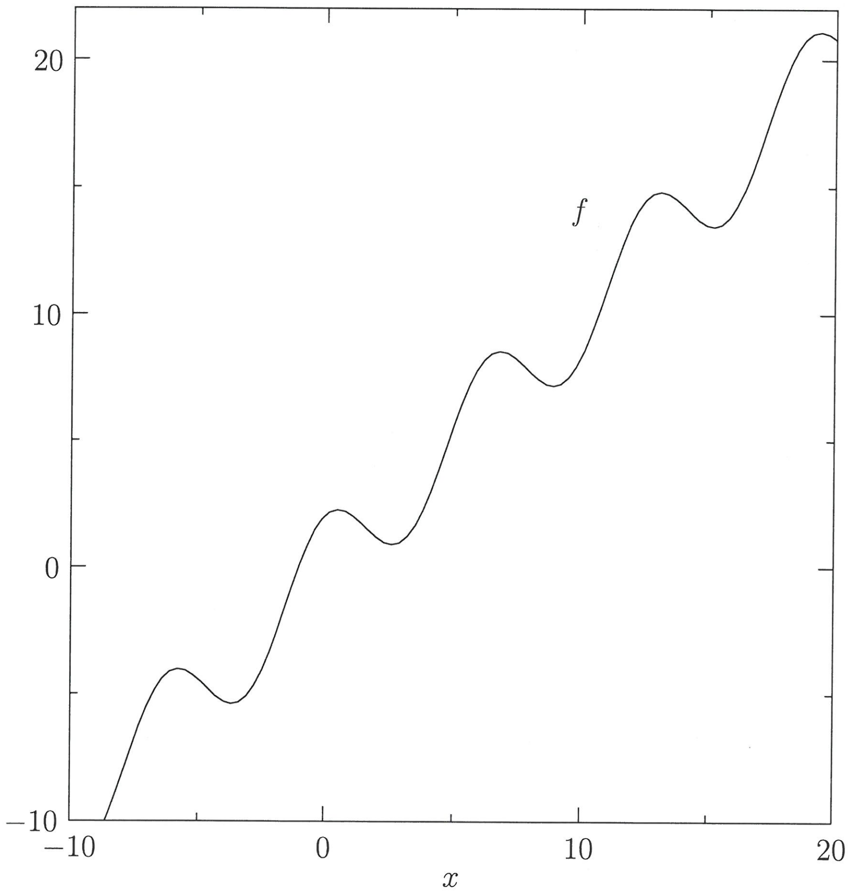


Figure 1: The graph of the function $f(x) = x + 2\cos(x)$ for $-10 \le x \le 20$.

Problem 3. Consider the graph of a function f in figure 2. Use the sheet on page 8 for your answers to the following:

3.a. (3 points) Indicate all points where $f'(x) = 0$.

3.b. (3 points) Sketch the graph of the derivative f'.

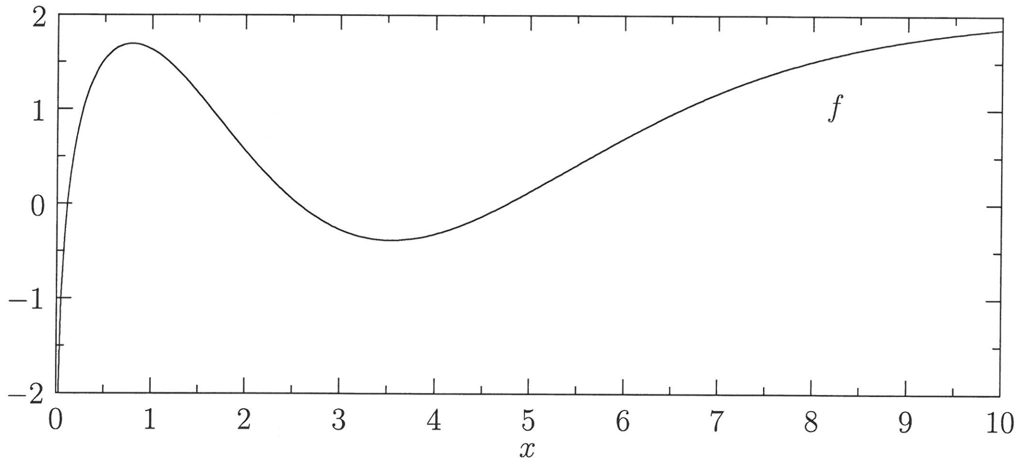


Figure 2: A function f.

Problem 4. Let $g(x) = e^{2x}$ where e is the base of the natural exponential function. Let

$$f(x) = \frac{\log_e(g(x)) - g'(x)}{2} + (e^x)^2.$$

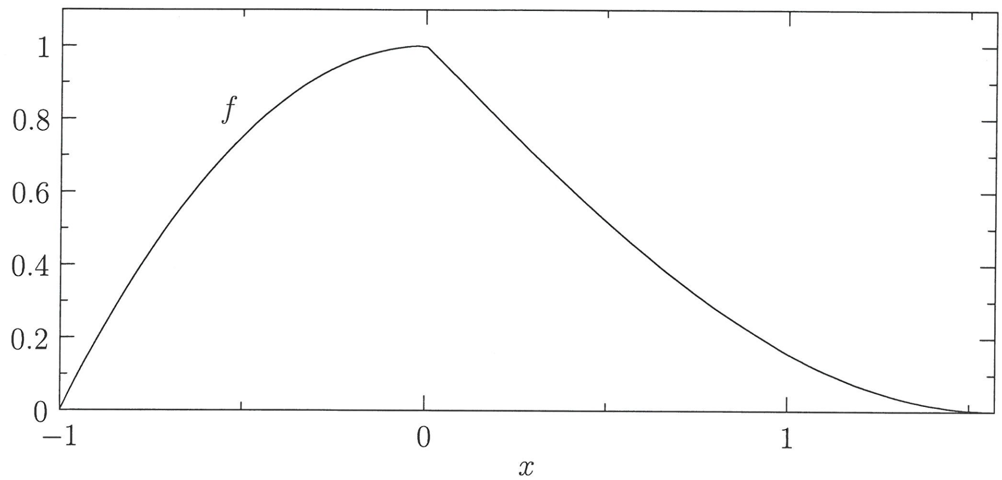
4.a. (3 points) Find a reduced form of the function f which does not depend on the function g.

$f(x) = x$

Problem 5. Let f be the function defined by

$$f(x) = \begin{cases} 1 - x^2, & x \in [-1, 0], \\ 1 - \sin(x), & x \in [0, \pi/2]. \end{cases}$$

The graph of f looks as follows:



5.a. (3 points) Compute $\int_{-1}^{0} f(x)\,dx$. $\frac{2}{3}$

5.b. (3 points) Compute $\int_{0}^{\pi/2} f(x)\,dx$. $\frac{\pi}{2} - 1$

5.c. (3 points) Find $\int_{-1}^{\pi/2} f(x)\,dx$. $\frac{\pi}{2} - \frac{1}{3}$

Problem 6. Let P, Q and R be three points in 3D-space; P has coordinates $(1, 2, 2)$, Q has coordinates $(1, 5, 2)$ and R has coordinates $(0, 3, 2)$.

6.a. (2 points) Find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

6.b. (3 points) Write parametric equations of the line L_1 that passes through P and Q and the line L_2 that passes through P and R.

6.c. (4 points) The lines L_1 and L_2 intersect at the point P. Find the angle between the two lines.

6.a. $\overrightarrow{PQ} = (0, 3, 0)$, $\overrightarrow{PR} = (-1, 1, 0)$

6.b. $(x, y, z) = (1, 2, 2) + t(0, 3, 0)$, $t \in \mathbb{R}$

$(x, y, z) = (1, 2, 2) + s(-1, 1, 0)$, $s \in \mathbb{R}$

6.c. $\frac{\pi}{4}$

Problem 7. Let P, Q and R be three points in 3D-space; P has coordinates $(2,1,5)$, Q has coordinates $(3,0,5)$ and R has coordinates $(3,1,7)$.

7.a. (2 points) Find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. $\overrightarrow{PQ} = (1,-1,0)$, $\overrightarrow{PR} = (1,0,2)$

7.b. (3 points) Compute the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$. $\overrightarrow{PQ} \times \overrightarrow{PR} = (-2,-2,1)$

7.c. (3 points) Find the area of the triangle with vertices P, Q and R. $\frac{3}{2}$

7.d. (3 points) Find an equation for the plane through P, Q and R.
$(-2)(x-2) + (-2)(y-1) + 1\cdot(z-5) = 0$

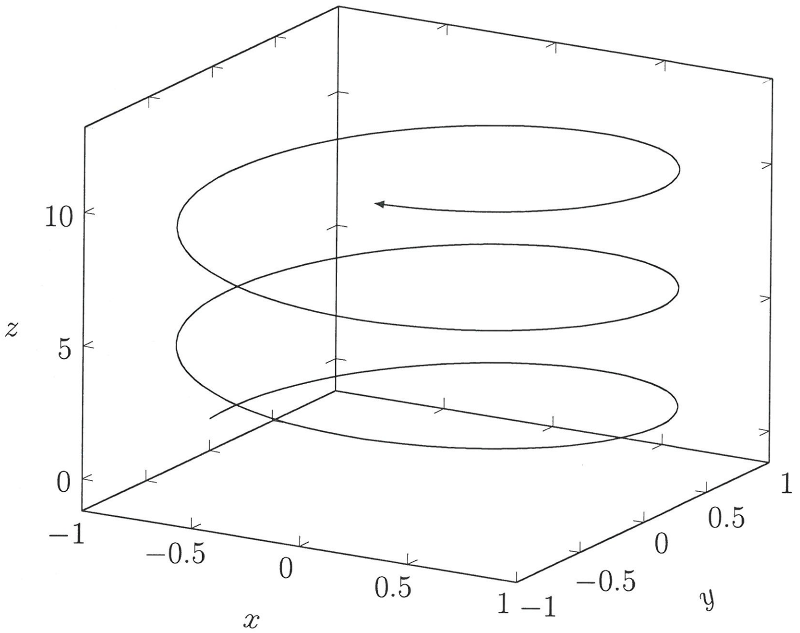
7.e. (2 points) Does the point $(3,-3,-1)$ lie on the plane? Why/why not?

Yes. Insert in the equation for the plane

Problem 8. A parametric curve is given by the following vector function:

$$\vec{r}(t) = (-\cos(t\sqrt{2}), \sin(t\sqrt{2}), t)$$

Here is a plot of the curve when t runs from 0 to 12:



8.a. (3 points) Show that every point on the curve lies on the cylinder described by $x^2 + y^2 = 1$. $(-\cos(t\sqrt{2}))^2 + (\sin(t\sqrt{2}))^2 = \cos^2(t\sqrt{2}) + \sin^2(t\sqrt{2}) = 1$

8.b. (2 points) Compute $\vec{r}'(t)$. $\vec{r}'(t) = (\sqrt{2}\sin(t\sqrt{2}), \sqrt{2}\cos(t\sqrt{2}), 1)$

8.c. (3 points) Find a t between 0 and 2π such that the velocity vector of the parametric curve equals $(1,1,1)$.

$t = \frac{\pi}{4\sqrt{2}}$

9.a.

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 3 \\ -2 & 6 & 4 & -2 \end{array}\right]$$

9.b.

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

9.c.

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 1 & -\frac{2}{3} & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

9.d.

$$\begin{cases} x_1 = 4 + 4x_3 \\ x_2 = 1 + \frac{2}{3}x_3 \\ x_3 \text{ free} \end{cases}$$

Problem 9. Consider the following system of linear equations:

$$\begin{aligned} x_1 - 4x_3 &= 4 \\ 3x_2 - 2x_3 &= 3 \\ -2x_1 + 6x_2 + 4x_3 &= -2 \end{aligned}$$

9.a. (2 points) Find the augmented matrix of the system.

9.b. (4 points) Find a row echelon form of the augmented matrix.

9.c. (3 points) Find the reduced row echelon form of the augmented matrix.

9.d. (4 points) Write down the general solution to the system.

9.e. (3 points) One solution to the system is $x_1 = 4$, $x_2 = 1$, $x_3 = 0$. Find another solution which has $x_3 = 3$.

9.f. (3 points) Find a solution to the system which has $x_1 = -8$.

9.e. $\begin{cases} x_1 = 16 \\ x_2 = 3 \\ x_3 = 3 \end{cases}$

9.f. $\begin{cases} x_1 = -8 \\ x_2 = -1 \\ x_3 = -3 \end{cases}$

Problem 10. Define ~~three~~ four matrices as follows:

$$A = \begin{bmatrix} 4 & 3 \\ 7 & -8 \\ -6 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 7 & -3 \\ 2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 3 \\ 1 & 1 \\ 2 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

10.a. (2 points) Find A^T.

$A^T = \begin{bmatrix} 4 & 7 & -6 \\ 3 & -8 & 2 \end{bmatrix}$

10.b. (3 points) Compute A^TC or CA^T.

$A^TC = \begin{bmatrix} -5 & 7 \\ -4 & 5 \end{bmatrix}$

10.c. (3 points) Compute $(C^TA)^T + B$.

$(C^TA)^T + B = \begin{bmatrix} 2 & 4 \\ -2 & 6 \end{bmatrix}$

10.d. (4 points) Find D^{-1}.

10.e. (2 points) Let $T(\vec{x}) = D\vec{x}$. Compute $T\left(\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}\right)$.

10.f. (3 points) Find an $\vec{x}$ such that $T(\vec{x}) = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.

10.d. $D^{-1} = \begin{bmatrix} -1 & -5 & 3 \\ 0 & 1 & 0 \\ 1 & 2 & -2 \end{bmatrix}$

10.e. $\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$

10.f. $\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

Appendix

Exact values of sin and cos for some angles:

- $\sin(\pi/6) = \cos(\pi/3) = 1/2$
- $\sin(\pi/4) = \cos(\pi/4) = 1/\sqrt{2} = \sqrt{2}/2$
- $\sin(\pi/3) = \cos(\pi/6) = \sqrt{3}/2$

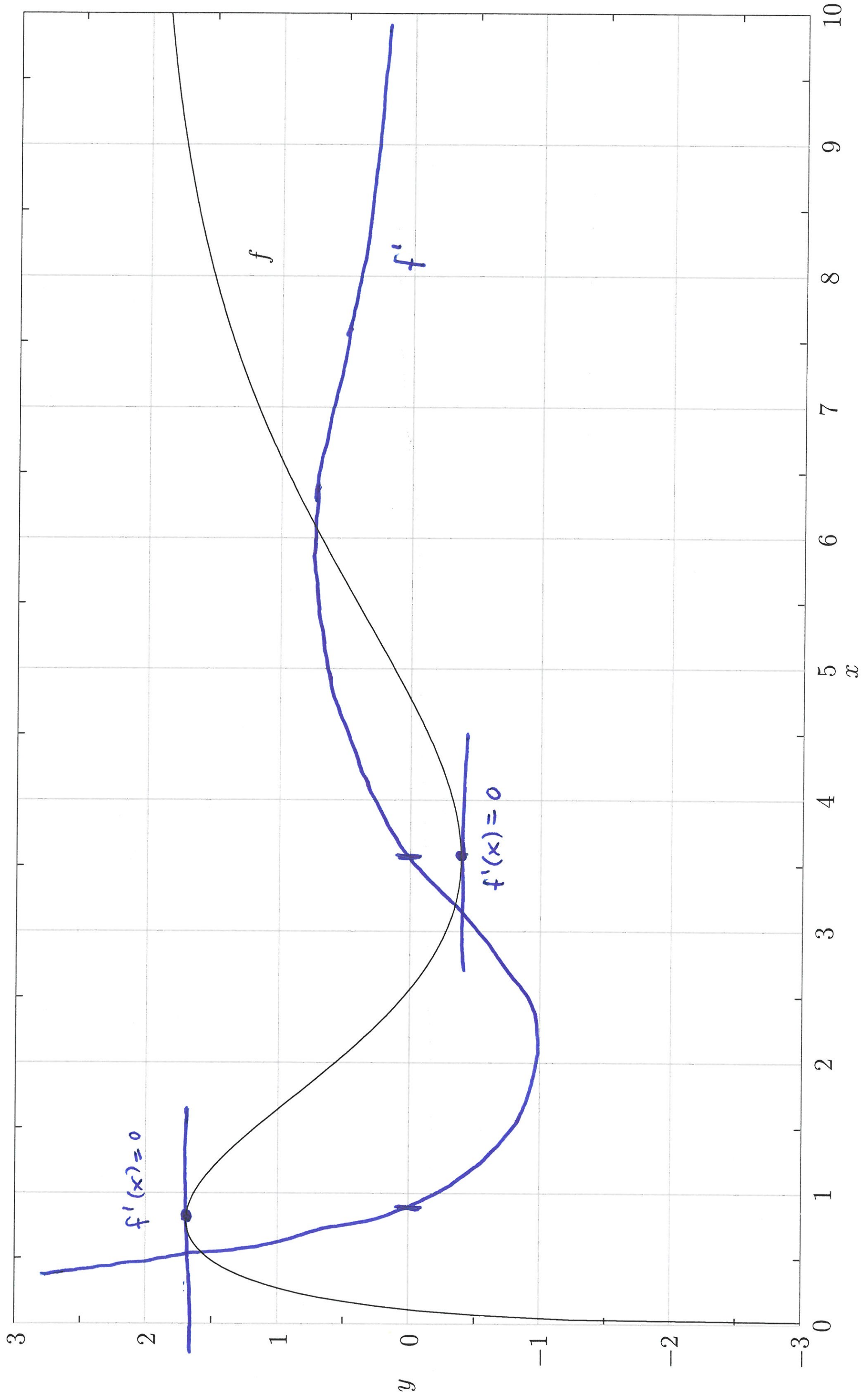


Figure 3: The function f