Massimo Ferri (Bologna)

Reduction and approximation of multidimensional persistent homology

The concept of “shape” has at least a geometrical, a topological and a psychological side. Persistent Topology proposes a formalization of it, which takes the three of them into account; it does it by studying the topological aspects of a filtration of a space, generally consisting of sublevel sets of a suitably chosen map (which we call “measuring function”). Main topological tool for this is homology, in the form of the “rank invariant” (RI).

While the case of a measuring function with R as range is fairly well understood, the case of a multidimensional range offers some difficulties. We expose and discuss a theorem which reduces the computation of the RI in the latter case to the former, by slicing the domain of the RI into particular half-planes.

Another important issue is how the RI of a given space relates to the one of a union of balls centered on a finite set of points sampling the space. We treat a recent result, which shows that there are well-determined regions of the domain of the RI, where the two coincide. For both items, we stress the implications for distances and shape comparison. Concepts exposed here will be further used and discussed in the talks by Claudia Landi and Patrizio Frosini.

Claudia Landi (Modena & Reggio Emilia)

Stability of Multidimensional Persistent Homology Groups

Persistent Topology is a theory for studying objects related to computer vision and computer graphics, by analyzing the qualitative and quantitative behavior of real-valued functions defined over topological spaces. To this aim, Persistence Topology studies the sequence of nested lower level sets of the considered functions and encodes at which scale a topological feature (e.g., a connected component, a tunnel, a void) is created, and when it is annihilated along this filtration.

In particular, multidimensional persistent homology captures the changes in the homology groups of a topological space filtrated by lower level sets of a vector-valued function, thus extending persistent homology theory to a multi-parameter setting.

Application of this theory to classification and retrieval of digital shapes requires stability with respect to perturbations in the studied object and in the considered filtration.

In this talk we present recent results assessing that multidimensional persistent homology behaves well in both these respects even under very mild assumptions on the considered spaces and functions.

More precisely, we consider the following questions: what conditions guarantee that multidimensional persistent homology groups are finitely generated? Which homology theory is better suited for the comparison of these groups? How to define a distance between rank invariants of these groups in order that small changes in the filtration induce small changes in this distance? How to use this distance in order to cope also with changes in the studied topological space? How is this related to more classical distances between sets?

We try to answer these questions starting from the results exposed in Massimo Ferri’s talk.

Eric Goubault (CEA List & Ecole Polytechnique)

Future Components and applications to concurrency theory

In this talk we will present the notion of "future-component category", which gives a meaning to retractions that preserve properties in the future of all points. We give some examples of computation (using maxplus polyhedra) and some applications. This is joint work with Emmanuel Haucourt and Sanjeevi Krishnan.

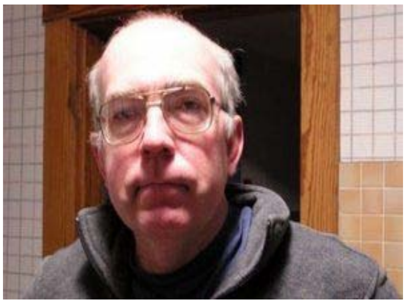
Emmanuel Haucourt (CEA List & Ecole Polytechnique)

Two equivalent ways of directing the spaces

The pospaces are the first mathematical objects that were considered in directed algebraic topology. Yet there are not satisfactory since they do not allow directed circles.
Inspired by the well-known notion of smooth manifold, L.Fajstrup, E.Goubault and M.Raussen have made a first attempt to go beyond this limitation by defining the local pospaces. However the properties of the category of local pospaces do not fit with some expected standard construction. Then S.Krishnan proposed an apparently slight (yet fundamental) modification to the definition of local pospaces, thus giving rise to what he has called the streams. In the meantime, M.Grandis has introduced the d-spaces.
Both d-spaces and streams are "natural", "intuitive" and their categories offer all the "good" properties one can expect, so it is not easy to decide which one should be preferred. A first step in this direction consists on remarking that there is a functor from Str (category of streams) to dTop (catetgory of d-spaces) which admits a left adjoint.

In this talk, we go further and prove that this pair of adjoint actually induces a pair of isomorphisms between a full subcategory of Str and a full subcategory of dTop, both of them being complete, cocomplete and even Cartesian closed if we restrict to weakly Hausdorff compactly generated spaces. These full subcategories still contain all the standard objects from directed topology (loops, vortex etc.). Moreover, this pair of isomorphisms is compatible both with the fundamental category construction and the directed geometric realization (in a sense that we will make precise).

In conclusion, d-spaces and streams are two equivalent and satisfactory ways of providing topological spaces with direction: working with one instead of the other is therefore just a matter of taste.



Maurice Herlihy (Brown)

The Index Lemma and Possible Sporadic Solutions to the Zero-One Exclusion Task

We consider a family of simple distributed coordination tasks, parameterized by an *n*-bit vector *v* where *n+1* is the number of processes. Processes are asynchronous. At the beginning, some subset of the processes wakes up. Those processes communicate with one another using a shared read-write memory, and eventually halt with a binary value. If $0 < k < n+1$ processes participate, they cannot all output *b[k].* If all processes participate, they cannot all output the same value.

We use the Index Lemma to derive a characteristic value for each instance of the family, with the property that the problem cannot be solved if the characteristic is non-zero. Curiously, the characteristic is non-zero "most of the time", but there are sporadic values for which it is zero. It remains an open question whether these instances of zero-one exclusion are solvable.

Joint work with Eli Gafni.

Armando Castañeda and Sergio Rajsbaum (UNAM)

Deriving an algorithm for the weak symmetry breaking task

In this talk we consider a distributed system consisting of n asynchronous processes identified 1, 2, …, n that communicate with each other using read and write operations to shared memory. Any number of them can fail by crashing. We focus on the weak symmetry breaking task (WSB), roughly specified as follows:
Each process starts with its own identifier as input value, and should eventually decide on a binary value, either 0 or 1. In every execution in which all processes participate, at least one process decides 0 and at least one process decides 1.
We have shown in the ACM PODC 2008 conference that for some values of n, WSB task is impossible and for the other exceptional values of n there exists a protocol that solves this task. These two results give new lower and upper bounds for the renaming task, which has been intensively studied for the last twenty years.

In this talk we describe combinatorial topology aspects of the upper bound. We present an algorithm that takes as input a binary colored subdivision of a n-simplex, such that the number of monochromatic (all vertices with the same binary color) n-simplexes counted by orientation is 0, and outputs a subdivision with the same boundary, but with zero monochromatic n-simplexes.
The techniques are based on Fan's formula, a generalization of both Sperner's and Tucker's lemma (a combinatorial version of the Borsuk-Ulam Theorem). A particular case of the formula is known as Index Lemma. The monochromatic simplex elimination algorithm described is reminiscent of the equivariant Hopf theorem.

Ulrich Fahrenberg (Aalborg)

History-preserving bisimilarity via open maps

Higher-dimensional automata represent a formalism in concurrency theory which allows for modeling any higher-order dependencies between events. In 2006, van Glabbeek showed that they provide a generalization to "[all] main models of concurrency proposed in the literature", up to history-preserving bisimilarity.

In this talk, we introduce two categories of higher-dimensional automata, a "strict" and a "homotopy" one. Both categories admit notions of open maps and bisimilarity, and we show that in the homotopy category, bisimilarity is precisely history-preserving bisimilarity. We concentrate on the unlabeled case.

The homotopy category builds on unfoldings of higher-dimensional automata, which are closely related to universal (directed) coverings, and we shall have a few things to say about this interplay between the combinatorics and geometry of higher-dimensional automata.

Lisbeth Fajstrup (Aalborg)

Structuring the category of d-spaces

A d-space is a topological space X with a subset P(X) of the set of paths X^I, denoted the dipaths. We require the subset to be closed under concatenation and reparametrization by increasing, not necessarily surjective maps f from I=[0,1] to itself .
The objects of the category dTop are d-spaces and morphisms are continuous maps f : Y → X with f(P(Y)) a subset of P(X).
In applications, we will often be happy with a much smaller category. E.g. subsets of R^n where dipaths are the coordinatewise increasing paths;(subsets of) products of directed graphs or cubical sets.
Given a topological space, the dipath structures, P(X) supported by the topology give rise to a lattice, where (X; P)≤ (X; Q) if the identity map on X is a d-map (X; P)→ (X; Q); i..e., if P is a subset of Q. This also defines a lattice structure on the d-structures, where we allow the topology to vary: (X; ρ; P)≤(X; σ; Q) if the identity map is a d-map. Here this means that σ is a subset of ρ and P is a subset of Q.
Given a particular d-structure P (such as the coordinate wise increasing paths on R^n), we get a lattice of sub-structures, namely the sub-lattice “below" P, i.e., we can restrict the pathologies allowed.
When we restrict X to be a subset of a product of digraphs with the product d-structure, i.e., X=Y-F where Y is the product of digraphs and F is a “forbidden area”, we describe the d-path structure on X as follows: P_F is the subset of dipaths on Y, which are “halted" on F, i.e., f is in P_F if *f(I) intersects F if and only if f is constant*.
There is a correspondence between the lattice structure and union and intersection of forbidden areas: Union of forbidden areas corresponds to intersection of d-structures, i.e., the meet in the lattice and intersection of forbidden areas corresponds to join in the lattice structure.
The lattice of d-structures on a topological space is complete. There is a top element, where all continuous paths are dipaths and a bottom where only constant paths are allowed.
Another useful hierarchy of the category of d-spaces is obtained from generated subcategories. Given a subcategory C of dTop, the subcategory $dTop_C$ generated by C is the full subcategory with objects d-spaces X, such that for all d-spaces Y, f : X→ Y is a d-map if and only if for is a d-map for all r: c→ X and all c in C. There is a hierarchy of generated subcategories in the sense that, if D is a subcategory of C, then $dTop_C$ is a full subcategory of $dTop_D$. Sufficient conditions for when the generated categories are equal have been studied in a recent paper. A useful example is that the category of directed cubes generates the same sub-category of dTop as the category containing the directed and the non-directed interval.

Martin Raussen (Aalborg)

(Prod-)Simplicial models for trace spaces

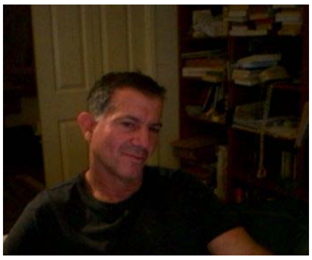
Concurrency theory in Computer Science studies the effects that arise when several processors run simultaneously sharing common resources. It attempts to advise methods to deal with e.g. ”the state space explosion problem”. In recent years, models with a combinatorial/topological flavor have been introduced and investigated as tools in the analysis of concurrent processes. It is a common feature of these models that an execution corresponds to a directed path (d-path), and that d-homotopies (preserving the directions) have equivalent computations as a result.
I will discuss a particular classical example of a directed space arising as a (pre-cubical set) model of a Higher Dimensional Automaton. For such a space, I will describe a new method yielding the homotopy type of the space of traces (executions) as a prodsimplicial complex – with products of simplices as building blocks. A description of that complex opens up for (machine) calculations of homology groups and other invariants from algebraic topology.
This prodsimplicial model arises from a covering of the trace space by particular contractible subspaces. Nonempty (and contractible) intersections of these subspaces form a poset category with a classifying space that corresponds to the barycentric subdivision of that prodsimplicial model.
The determination of the model depends on deciding whether certain subspaces of traces are empty are not. That decision relies on (a simple version of) an algorithm detecting deadlocks and unsafe regions for these Higher Dimensional Automata by checking a lot of inequalities. The last step requires finding minimal transversals in a (potentially huge) hypergraph, a problem that is studied in the combinatorics literature.

Patrizio Frosini (Bologna)

Multidimensional persistent topology as a metric approach to shape comparison

In this talk we motivate the central role of the so-called *natural pseudodistance* in multidimensional persistent topology, and present some new results concerning the computation of lower bounds for this pseudodistance. We start noting that, however shape can be defined, it is based on stable perceptions made by observers, at least in an empirical setting. This dependence on the observers follows from the large subjectivity we experience in shape comparison, while stability is requested by the fact that human judgments focus on persistent properties of the real world, while non-persistent properties are usually interpreted as noise.
In order to express stability in a mathematical setting we need to model the set of observations as a topological space *T*, while the observer's perception can often be seen as a function φ taking each observation $t \, \varepsilon T$ to a vector in $\boldsymbol{R}^n$.
This function φ describes *t* from the point of view of the observer. When two pairs (T_1, φ_1), (T_2, φ_2) are chosen for "comparable perceptions", it is natural to consider the functional θ taking each homeomorphism *h*: *T*1 → *T*2 to the L_∞-norm of the function φ_1-$\varphi_2 \circ h$. This functional represents the "cost" of the matching between perceptions induced by *h*. The lower this cost, the better the matching between the two perceptions is. The natural pseudodistance *d* between the pairs (T_1, φ_1), (T_2, φ_2) is just the infimum of this cost θ(*h*), varying *h*.
Lower bounds for *d* can be obtained by computing multidimensional size functions, size homotopy groups and persistent homology groups for the pairs (T_1, φ_1), (T_2, φ_2). This fact motivates the second part of this talk, where we illustrate two new results following by the foliation method in multidimensional persistent topology: a localization theorem for the discontinuities of multidimensional size functions and a result allowing to obtain arbitrarily good approximations of the matching distance between multidimensional rank invariants.
This talk will be strongly linked to the ones given by Massimo Ferri and Claudia Landi.

Eli Gafni (UCLA)



Bigamy: Intermingling Algebraic-Topology and Distributed-Algorithms

Three papers in 1993 opened the way to the discovery of the duality between distributed-algorithms and algebraic-topology: A solution to a distributed task exist if and only if simplicial chromatic map between certain topological complexes exists. Since then the standard use of this duality is to transform a distributed problem into a topological problem, and then exclusively use topological machinery.
But ”duality” is a 2-way street: A map exists if certain task is solvable. In this talk I will take results and show how hard they are to derive exclusively topologically, while how easy and convincing the derivation becomes when combining the machineries.

In particular, I will derive the simplicial approximation theorem of algebraic topology using solely algorithmic reasoning, as well as show that all sort of impossibility results which a-posteriori can be derived through Sperner-Lemma have easy derivation using the notion of read-write reduction.

Michael Farber (Durham)

Stochastic algebraic topology in robotics

In my talk I will describe solutions to several problems of mixed probabilistic – topological nature which are inspired by applications in topological robotics. These problems deal with systems depending on a large number $(n\to\infty)$ of random parameters. Our results show that various topological characteristics of configuration spaces of such systems can be predicted with high confidence.

Robert Ghrist (Penn)

Euler Calculus for Data

This talk covers an ingenious integral calculus based on Euler characteristic, stemming from work on sheaves due to MacPherson and Kashiwara in the 1970s, and connecting back further to classical integral geometry. I will emphasize its novel utility in data management, particularly in aggregation of redundant data and inverse problems over sensor networks.
Applications to target counting and localization will be given. These applications motivate the investigation of a new branch of numerical integration theory.

Michael Robinson (Penn)

"Constructible sheaves and their cohomology for asynchronous logic and computation"

Sheaf theory, the descriptive branch of topology that examines how local algebraic structure assembles into global structure, has been a useful tool for some fifty years in pure mathematics. It has a reputation for being technical, subtle, and not terribly applicable outside of pure mathematics.
Recently, sheaf theory has been examined in the context of o-minimal structures, which place strong bounds on the complexity of the base topological space and the sheaves thatcan be assembled onto it. In particular, the theory of constructible sheaves permits one to more clearly see how to assemble sheaves for doing specific *applied* tasks. Better, due to advances in computational topology, the Čech cohomology of these sheaves can be computed directly.

I will describe some of our recent work in sheaf theory, as it casts certain sufficiency conditions in computing and logic as the presence or absence of certain cohomology classes for a specific kind of constructible sheaf and its direct images. In particular, sections of these sheaves capture in a precise fashion the state of a logical system. They provide way to understand how (even momentary) storage of information arises even when there is no such storage on a local level.
This gives a principled way to approach difficult design problems in asynchronous logic design and in asynchronous computation generally.

Peter Bubenik (Cleveland)

Concurrency and quasi-categories

Topologists often replace topological spaces with simplicial sets. The latter can be thought of as a combinatorial model for the former that is more amenable to calculations, and the two are equivalent in a precise sense. Concurrent systems can be modeled using directed spaces. I will describe work in progress, joint with David Spivak, on giving a directed version of the passage from topological spaces to simplicial sets.

Philippe Gaucher (Paris 7)

About higher dimensional transition systems

I will introduce the category of weak higher dimensional transition systems which contains the Cattani-Sassone ones as a small-orthogonality class. I will explain its categorical and homotopical properties, and the links with process algebras, bisimulation, and the topological models of concurrency.

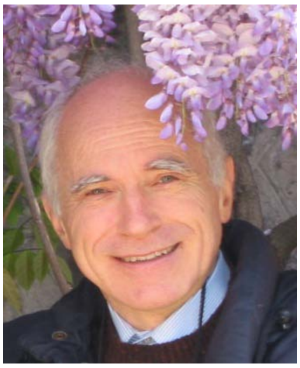
Dmitry Feichtner-Kozlov (Bremen)

Stellar equipartitions

In this talk we consider the following problem: given a geometric *d*-simplex Δ and the set *S* of *n* points in the interior of Δ find a stellar subdivision of Δ, such that the interiors of all the *d*-simplices of that subdivision contain equally many points from *S*.

In general, such a stellar subdivision need not exist. However, if the points of *S* are in sufficiently general position with respect to the simplices of Δ, then we prove that such a stellar subdivision in fact does exist, as well as show how to find its center.

We will apply these results to the problem of dynamically maintaining a balanced system of master sensors in a sensor network.



Marco Grandis (Genova)

Directed algebraic topology and symmetries

Formal settings for directed algebraic topology have been developed in my recent book on Directed Algebraic Topology (Cambridge U.P. 2009). They are based on an abstract *cylinder functor I(X)* and natural transformations between its powers, like faces $\delta^{\pm}$: 1→*I*), degeneracy (*e: I* → 1) and connections ($g^{\pm}$: I^2 → *I*). Or, dually, on a *cocylinder functor* P(Y), representing the object of (directed) paths of an object Y. Or also, on an adjunction between *I* and *P* that allows one to see (directed) homotopies of maps as morphisms *I(X)* → Y or equivalently *X* → *P(Y);* then, the natural transformations between powers of *I* correspond to their mates, natural transformations between powers of *P*.

In the classical case, settings based on the cylinder (or path) endofunctor go back to Kan's well-known papers on 'abstract homotopy'. Quillen model structures seem to be less suited to formalize directed homotopy.

As a crucial fact, the present formalization deals with endofunctors and `operations' on them. It is thus `categorically algebraic', like the theory of monads, a classical tool of category theory. This is why such settings can generally be lifted from a ground category to categorical constructions on the latter, like categories of diagrams, or sheaves, or algebras for a monad.

An important role is played by symmetries. In the classical (non-directed) case of topological spaces, these are generated by two basic instances, the reversion and the transposition

r: I(X) → I(X); r(x; t) = (x; 1 - t);

s: I^2*(X) → I2(X); s(x; t1; t2) = (x; t2; t1);*

(or their mates, *r: P→P and s:* P^2 → P^2). In directed algebraic topology, the first kind of symmetry must be broken and the second can.

Indeed, in all categories of `directed objects' there is an involutive covariant endofunctor *R*, called reversor, which turns a directed object into the opposite one; its action on preordered spaces, d-spaces and categories is obvious; for cubical sets, one interchanges lower and upper faces. Then, the ordinary reversion is replaced with a reflection, in the opposite directed space: a natural transformation *r: IR→RI (or r: RP → PR)*. Notice that the reversible case is a particular instance of the directed one, where *R* is the identity functor.

On the other hand, a transposition exists in various directed structures, for instance preordered spaces and d-spaces, but does not exist in other cases, e.g. for cubical sets. On the one hand, its presence yields important consequences, like the homotopy invariance of the cylinder and path functors. On the other hand, it restricts the interest of directed homology and prevents a good relation of the latter with suspension.

Relative settings, formed of a basic setting equipped with a forgetful functor with values in a strong setting, allow us to handle the defective structures.

Rodolfo Conde Martinez and Sergio Rajsbaum (UNAM)

The iterated shared memory model of computation and an enrichment with safe consensus tasks

The *Iterated Snapshot* model is an asynchronous computation model where the processes communicate through a sequence of snapshot objects. Each process accesses each snapshot object only once-- writes a value to its register, and gets a snapshot of its contents.
Processes access the sequence of objects, one-by-one, in the same order, and asynchronously.
Any number of processes can crash.
It is known that this model is equivalent to the usual wait-free read/write shared memory model, but its runs are more structured and easier to analyze than the runs in the shared memory model.
Extensions of the model have also been studied, where the processes communicate through a sequence of objects more powerful than snapshot objects.

In this talk we introduce this research area to the audience, and describe a new extension of the model, with the *safe consensus* objects of Afek, Gafni and Lieber. In a safe consensus object, the validity condition of consensus is weakened as follows: if the 1st process to invoke the object returns before any other process invokes it, then it outputs its input; otherwise the consensus output can be arbitrary.

Sanjeevi Krishnan (U.S. Naval Lab)

THE SPACETIME GEOMETRY OF STRING REWRITING

String rewriting systems can serve as toy models for protein folding, theorem-proving, and more general computational processes; the monoids presented by such rewriting systems represent the underlying "logics" of these processes. On a monoid M, the literature presents a number of homotopical invariants, on spaces associated to M, which can rule out the existence of finite and complete string rewriting presentations of M - particular solutions to the word problem on M. We will first see how the transition from classical topology to directed topology, the study of spaces equipped with temporal structure, allows us to extend these results through the use of new directed homological invariants. We will then discuss potential applications to protein-folding. This talk assumes no prior experience with string rewriting, homology, or homotopy theory.

Samuel Mimram (CEA List & Ecole Polytechnique)

Cubical sets and Petri nets: an adjunction

Petri nets are a widely used model for concurrent processes, which focuses on expressing the resources needed and the results produced by events occurring in computations. This model was related by Winskel and Nielsen to asynchronous transition systems which, in the spirit of true concurrency, represent two computations done in parallel as a family of equivalent interleavings of these computations. We extend here their result into an adjunction between Petri nets and cubical sets, in order to take higher-order commutations of events in account.