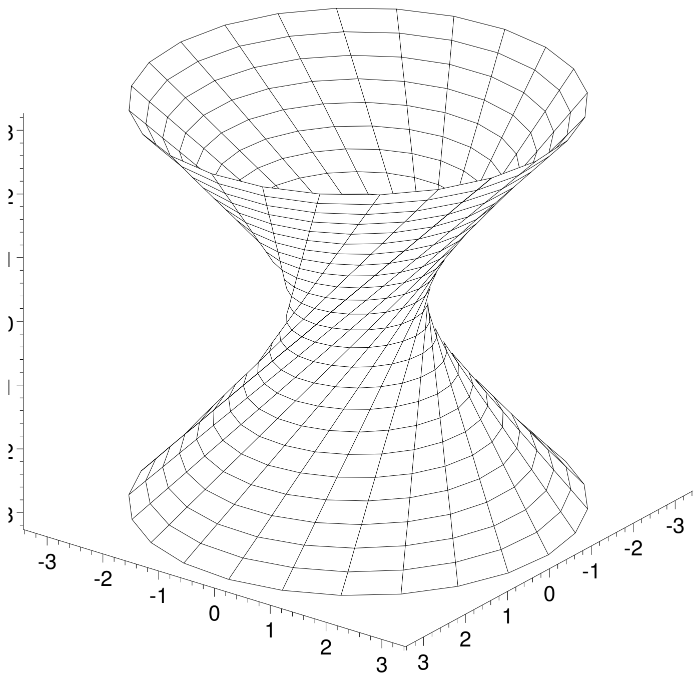
Løsningsskitse:

10.1 Se Fig. 1.



Figur 1: En omdrejningshyperboloide

1. Indsæt parameterfremstillingen
$$[x, y, z] = \mathbf{r}(u, v) = [\cos v - u \sin v, \sin v + u \cos v, u]$$
i $x^2 + y^2 - z^2$. Snittet af fladen med planen $z = a$ er en cirkel i denne plan med centrum i $(0, 0, a)$ og radius $\sqrt{1 + a^2}$.
2. $\mathbf{r}_u(u, v) = [-\sin v, \cos v, 1]$, $\mathbf{r}_v(u, v) = [-\sin v - u \cos v, \cos v - u \sin v, 0]$, $E = 2,\ F = 1,\ G = 1 + u^2$.
3. $(\mathbf{r}_u \times \mathbf{r}_v)(u, v) = [u \sin v - \cos v, -u \cos v - \sin v, u]$, dvs., for P med $\overrightarrow{0P} = [x_0, y_0, z_0] = \mathbf{r}(u_0, v_0)$ gælder: $\mathbf{n}_P = (\mathbf{r}_u \times \mathbf{r}_v)(u_0, v_0) = [-x_0, -y_0, z_0]$ er normal til tangentplanen i P.
Alternativ løsning: Gradienten til $f : \mathbf{R}^3 \to \mathbf{R}$, $f(x, y, z) = x^2 + y^2 - z^2$ i $[x_0, y_0, z_0]$, $\nabla f(x_0, y_0, z_0) = (2x_0, 2y_0, -2z_0)$, er normal til fladen (=niveauflade $f(x, y, z) = 1$).
Ligning for tangentplan: $-x_0 x - y_0 y + z_0 z + 1 = 0$.
4. $\int_0^{2\pi} \int_{-1}^{1} \sqrt{EG - F^2} du dv = \int_0^{2\pi} \int_{-1} \sqrt{2u^2 + 1} du dv = 2\sqrt{2}\pi \int_{-1}^{1} \sqrt{u^2 + \frac{1}{2}} du = (2\sqrt{3} + \frac{\sqrt{2}}{2} \ln(5 + 2\sqrt{6}))\pi \cong 16.$

5. $\mathbf{r}_{uu}(u,v) = \mathbf{0}$, $\mathbf{r}_{uv}(u,v) = [-\cos v, -\sin v, 0]$,
$\mathbf{r}_{vv}(u,v) = [-\cos v + u\sin v, -\sin v - u\cos v, 0]$,
$e = 0$, $f(u,v) = \frac{1}{\sqrt{2u^2+1}}$, $g(u,v) = \frac{u^2+1}{\sqrt{2u^2+1}}$.

$$K(u,v) = \frac{eg-f^2}{EG-F^2}(u,v) = \frac{-1}{(2u^2+1)^2} < 0,$$

$$H(u,v) = \frac{Eg-2Ff+Ge}{2(EG-F^2)}(u,v) = \frac{u^2}{(2u^2+1)^{\frac{3}{2}}}.$$

6.
$$k_1(u,v) = (H + \sqrt{H^2-K})(u,v) = \frac{1}{(2u^2+1)^{\frac{1}{2}}},$$

$$k_2(u,v) = (H - \sqrt{H^2-K})(u,v) = \frac{-1}{(2u^2+1)^{\frac{3}{2}}}.$$

10.2 $\mathbf{r}_u(u,v) = [2u,0,1]$, $\mathbf{r}_v(u,v) = [0,2v,1]$, $(\mathbf{r}_u \times \mathbf{r}_v)(u,v) = [-2v,-2u,4uv] = 2[-v,-u,2uv]$, $\mathbf{r}_{uu}(u,v) = [2,0,0]$, $\mathbf{r}_{uv}(u,v) = \mathbf{0}$, $\mathbf{r}_{vv}(u,v) = [0,2,0]$.

1. Vektoren $\mathbf{n} = \frac{1}{2}(\mathbf{r}_u \times \mathbf{r}_v)(1,1) = [-1,-1,2]$ er normalvektor i P_1. Ligning for tangentplanen: $0 = \mathbf{n} \cdot ([x,y,z] - \overrightarrow{OP_1}) = [-1,-1,2] \cdot [x-1, y-1, z-2] = -(x-1) - (y-1) + 2(z-2) = -x - y + 2z - 2$.
2. $E(u,v) = \mathbf{r}_u(u,v) \cdot \mathbf{r}_u(u,v) = [2u,0,1] \cdot [2u,0,1] = 4u^2+1$,
$F(u,v) = \mathbf{r}_u(u,v) \cdot \mathbf{r}_v(u,v) = [2u,0,1] \cdot [0,2v,1] = 1$,
$G(u,v) = \mathbf{r}_v(u,v) \cdot \mathbf{r}_v(u,v) = [0,2v,1] \cdot [0,2v,1] = 4v^2+1$.
$e(u,v) = \frac{\mathbf{r}_{uu}(u,v)\cdot(\mathbf{r}_u \times \mathbf{r}_v)(u,v)}{\|(\mathbf{r}_u \times \mathbf{r}_v)(u,v)\|} = \frac{[2,0,0]\cdot[-v,-u,2uv]}{\sqrt{u^2+v^2+4u^2v^2}} = \frac{-2v}{\sqrt{u^2+v^2+4u^2v^2}}$.
$f(u,v) = 0$. $g(u,v) = \frac{-2u}{\sqrt{u^2+v^2+4u^2v^2}}$.
3. $K(u,v) = \frac{eg-f^2}{EG-F^2}(u,v) = \frac{4uv}{4(u^2+v^2+4u^2v^2)} = \frac{uv}{(u^2+v^2+4u^2v^2)}$.
4. For $v > 0$ er P_{uv} elliptisk ($K > 0$), for $v < 0$ hyperbolsk ($K < 0$). På figuren er punkterne over den hvide parabel elliptiske og under den er de hyperbolske (sadelpunkter).